FE Review – Basic Circuits

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FE topics

General FE

14. Electricity, Power, and Magnetism 7–11
A. Electrical fundamentals (e.g., charge, current, voltage, resistance, power, energy)
B. Current and voltage laws (Kirchhoff, Ohm)
C. DC circuits
D. Equivalent circuits (series, parallel, Norton's theorem, Thevenin's theorem)
E. Capacitance and inductance
F. AC circuits (e.g., real and imaginary components, complex numbers, power factor, reactance and impedance)
G. Measuring devices (e.g., voltmeter, ammeter, wattmeter)

Mechanical FE

Electricity and Magnetism 3–5
A. Charge, current, voltage, power, and energy
B. Current and voltage laws (Kirchhoff, Ohm)
C. Equivalent circuits (series, parallel)
D. AC circuits
E. Motors and generators

Industrial FE

Engineering Sciences 5–8
A. Work, energy, and power
B. Material properties and selection
C. Charge, energy, current, voltage, and power

Chemical FE

Engineering Sciences 4–6
A. Applications of vector analysis (e.g., statics)
B. Basic dynamics (e.g., friction, force, mass, acceleration, momentum)
C. Work, energy, and power (as applied to particles or rigid bodies)
D. Electricity and current and voltage laws (e.g., charge, energy, current, voltage, power, Kirchhoff, Ohm)

Topics to Be Covered

- DC Circuits and Analysis
  - DC Voltage
  - DC Current
  - Power
  - Resistance
  - Resistivity
  - Ohms Law
  - Series and Parallel Combinations of Resistors
  - Voltage and Current Dividers
  - Capacitors
  - Inductors
  - Kirchhoff's Laws
  - Simple Resistive Circuit Solving
  - Delta-Wye Transforms
  - Superposition
  - Source Transforms and Thevenin/Norton Equivalent Circuits
  - Maximum Power Transfer
  - RC and LC Transients
- AC Circuits
  - AC Waveforms
  - Sine and Cosine Relations
  - RMS Values
  - Phasor Transforms
  - Phase Angles
  - Impedance
  - Ohm's Law for AC circuits
  - Complex Power
  - Resonance
  - Transformers

DC Circuits and Analysis

Definition of Voltage

- Separating electric charge requires energy. The potential energy per unit charge created by charge separation is VOLTAGE. It is often referred to as electrical potential.

$$v = \frac{dw}{dq}$$

where v = voltage in volts,

w = energy in joules

q = charge in coulombs

It is a Δ quantity… i.e. measured at one point relative to another… most typically relative to the electrical potential of the Earth (ground).

Definition of Current

- Flowing charges create electrical effects. The rate of charge flow is called CURRENT

$$i = \frac{dq}{dt}$$

where i = current in amperes,

q = charge in coulombs

t = time in seconds

We typically think of current flowing through wires in our circuits much like water through pipes

Electrical Power

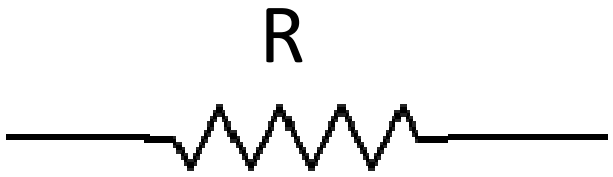
- The power supplied or dissipated in a circuit element can be found from the following relation:

$$P = iv$$

- A negative power denotes power being supplied by the element, positive power denotes power being dissipated by the element

Resistance

- Resistance is the ability of a circuit element to impede current flow. It is denoted by R.



- Resistance is measured in Ohms (Ω). Energy is dissipated in a resistor and converted into heat.

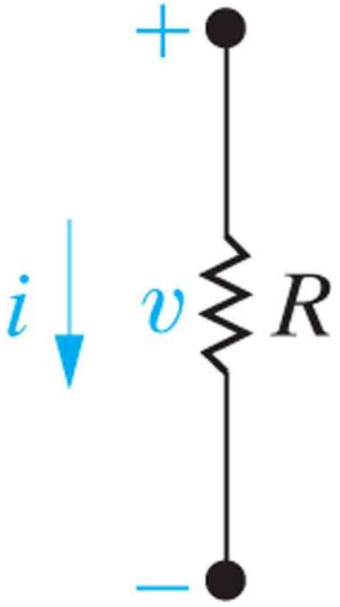
Resistivity

- Resistivity (ρ) is a material property that governs resistance. The resistivity of a material along with its physical geometry determine its resistance. It is defined in terms of units of (Ohm * meters)

$$R = \rho \frac{l}{A}$$

Ohm's Law

- Ohm's Law provides a relationship between the voltage dropped across a resistive element and the current flowing through it.



$$v = iR$$

By convention, the ohm's law voltage drops in the direction of current flow

Power dissipated in a Resistor

Recall that: Power = v*i

Using Ohm's Law, the power dissipated in a resistor can be re-written multiple ways:

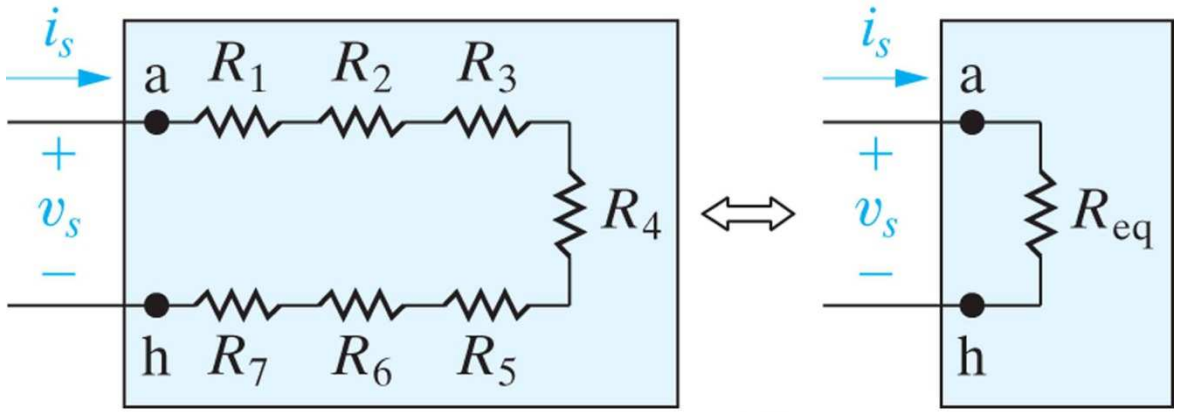
$$P = (i*R)*i = i^2R$$

$$P = v*(v/R) = v^2/R$$

These expressions hold true regardless of sign convention… Power is always dissipated in a resistor.

Combinations of Resistors

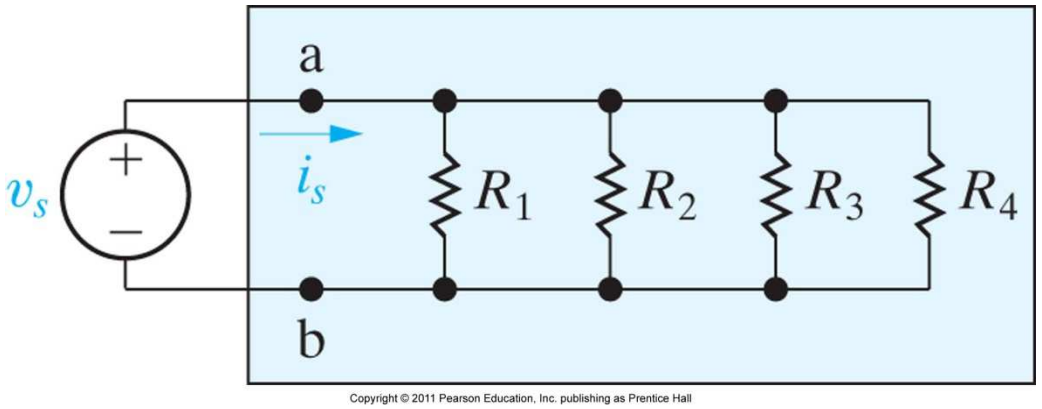
- Separate Resistors in a circuit can be combined to create a simplified equivalent resistance
- Resistors in Series
  - When resistors are connected in series (end to end) they can simply be added



$$R_{eq} = R_1 + R_2 + R_3 + R_4 + \ldots$$

Combinations of Resistors

- Resistors in Parallel
  - When resistors are connected in Parallel (side to side) they add in the inverse



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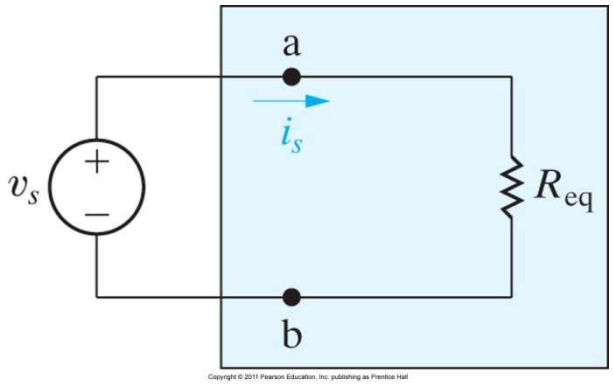
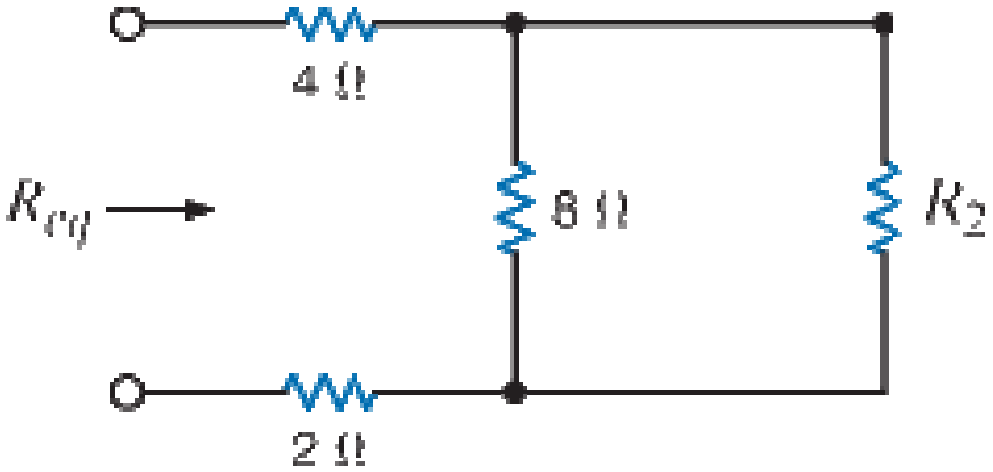


$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} + \ldots$$

Example FE Problem

If R_{eq}=10.8Ω, what is R_2?



a) 12Ω
b) 20Ω
c) 8Ω
d) 18Ω

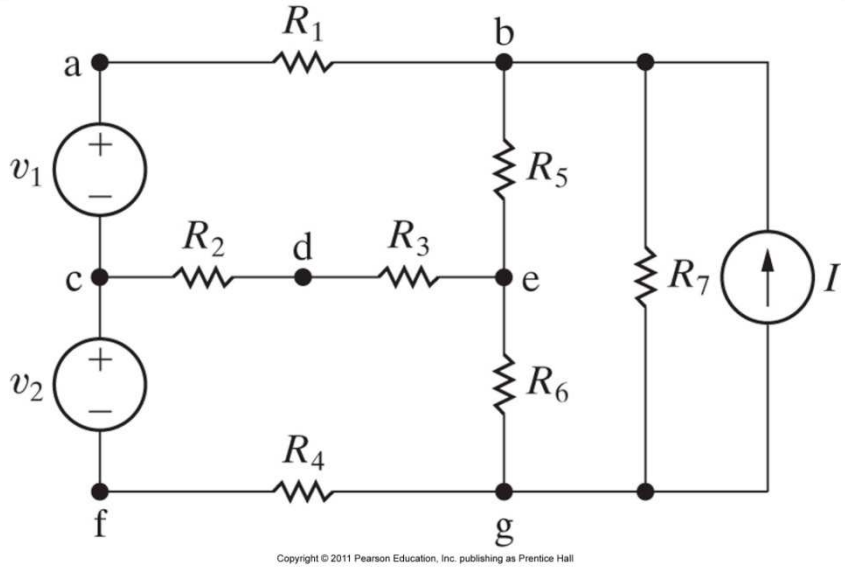
Solving DC Circuits

- We have covered voltage and current (the two parameters we wish to find throughout an electrical circuit) as well as basic circuit elements.
- This allows us to begin to construct and model basic resistive circuits
- We will need some basic circuit theorems in order to solve these circuits.

Circuit Terminology

TABLE 4.1 Terms for Describing Circuits

Name	Definition	Example From Fig. 4.3
node	A point where two or more circuit elements join	a
essential node	A node where three or more circuit elements join	b
path	A trace of adjoining basic elements with no elements included more than once	$v_1 - R_1 - R_5 - R_6$
branch	A path that connects two nodes	R_1
essential branch	A path which connects two essential nodes without passing through an essential node	$v_1 - R_1$
loop	A path whose last node is the same as the starting node	$v_1 - R_1 - R_5 - R_6 - R_4 - v_2$
mesh	A loop that does not enclose any other loops	$v_1 - R_1 - R_5 - R_3 - R_2$
planar circuit	A circuit that can be drawn on a plane with no crossing branches	Fig. 4.3 is a planar circuit Fig. 4.2 is a nonplanar circuit



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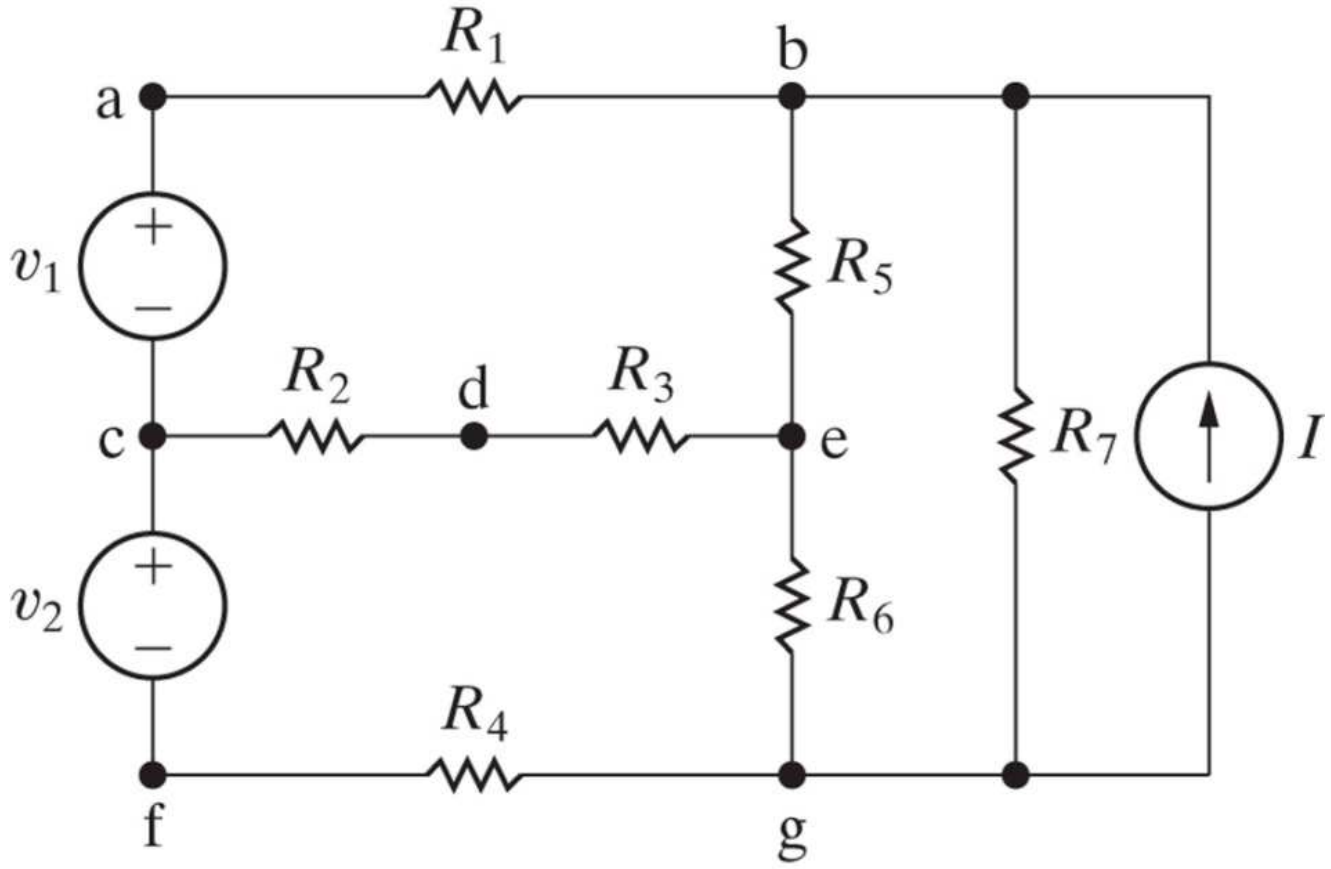
Practice



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How Many Nodes?

How Many Essential Nodes?

How Many Branches?

How Many Essential Branches?

How Many Meshes?

Kirchoff's Current Law

- Kirchoff's Current Law (KCL)
  - **The algebraic sum of all the currents at any node in a circuit equals zero**
  - This means that the net current entering a node is equal to the net current exiting the node… no charge is stored in the node.
- To use KCL you must assign a sign to use for all current entering the node, and an opposite sign to all current exiting the node. All currents entering and exiting the node are then summed to zero.

Kirchoff's Voltage Law

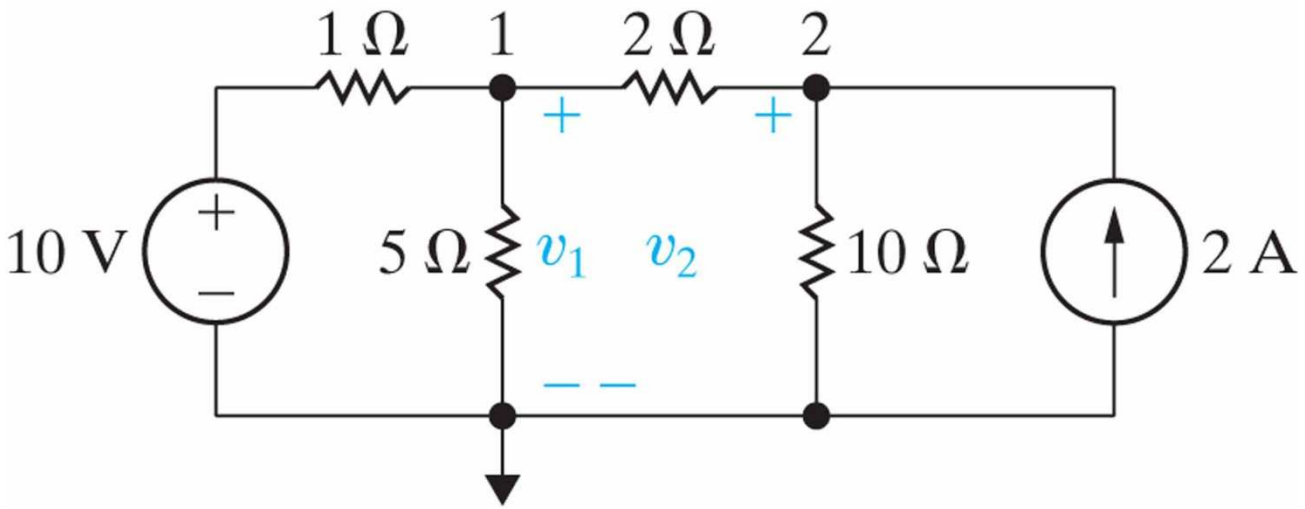
- Kirchoff's Voltage Law (KVL)
  - **The algebraic sum of all the voltage drops around any closed path in a circuit is equal to zero**
  - A closed path in a circuit is a path that takes you from a starting node back to that node without passing through any intermediate node more than once
  - To use KVL, simply find a closed path and sum the voltage drops you encounter and set the sum equal to zero

Systematic Approach

- By immediately identifying the essential nodes and branches of a circuit, we can immediately go about systematically solving the circuit.
- Using KVL and KCL on essential nodes and branches is a perfectly good and valid technique for circuit solving
- However, there are new variables called node voltages and mesh currents that let us further reduce the number of equations needed for solving a circuit

Node-Voltage Method

- 1st Steps in Node-Voltage Method
  - Make a neat layout of your circuit with no branches crossing over
  - Mark the essential nodes
  - Pick one of the essential nodes to be your reference node and mark it. (typically the node with the most branches)
  - Define the node voltages (voltages from reference node to other essential nodes)



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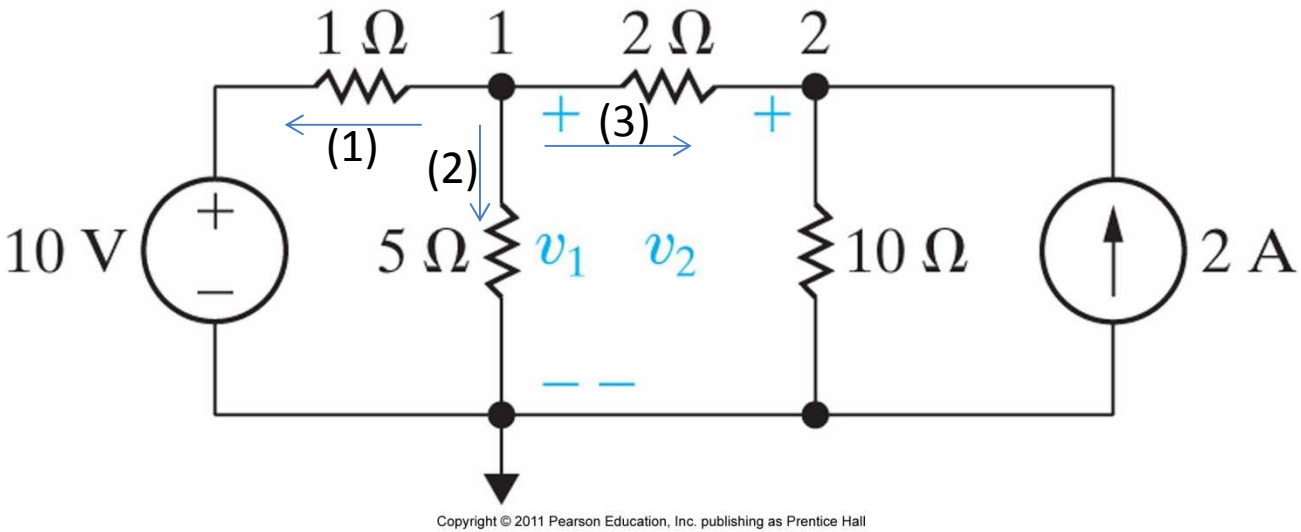
Node-Voltage Method Continued

- With the node voltages labeled in your circuit you are ready to generate your node-voltage equations
  - This is done by writing each of the essential branch currents leaving the essential nodes in terms of the node voltages and then summing them to zero in accordance with Kirchoff's current law.

Example of N-V method



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There are three branch currents coming out of node 1:

The current (1) through the 1Ω resistor and 10V source

The current (2) through the 5Ω resistor

The current (3) through the 2Ω resistor

These can be written in terms of the node voltages, v_1 and v_2.

$$\frac{v_1 - 10\ V}{1\Omega} + \frac{v_1}{5\,\Omega} + \frac{v_1 - v_2}{2\,\Omega} = 0$$

Finishing Example

- This same process can be used to produce the N-V equation for node 2.

$$\frac{v_2 - v_1}{2} + \frac{v_2}{10} - 2 = 0$$

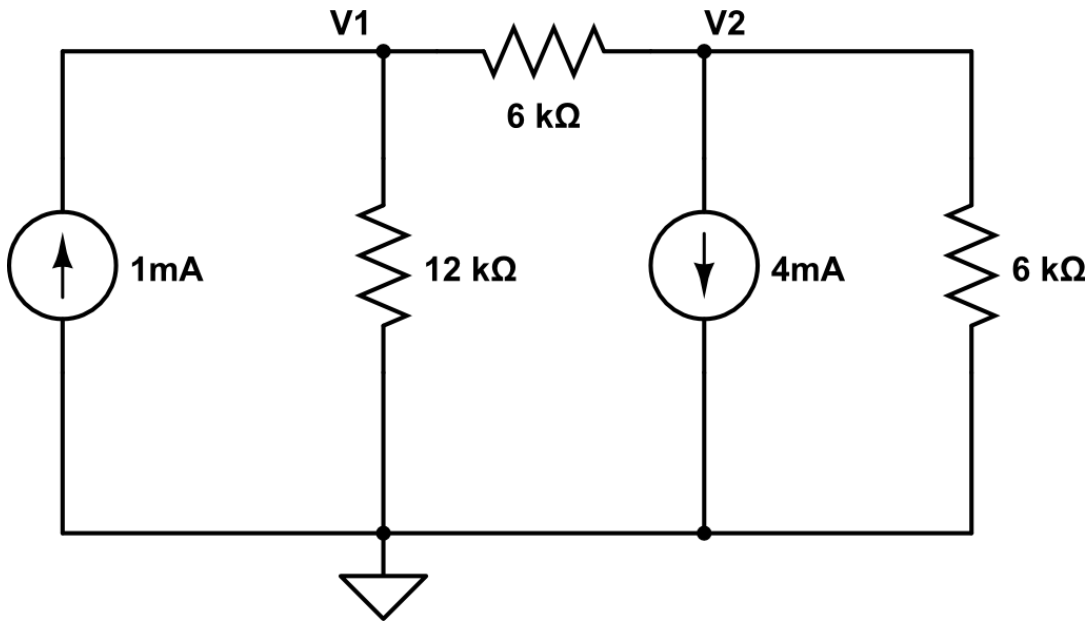
- These two equations have two unknowns, v_1 and v_2, thus the two node voltages can be calculated and the branch currents determined from them.

$$v_1 = 9.09V$$

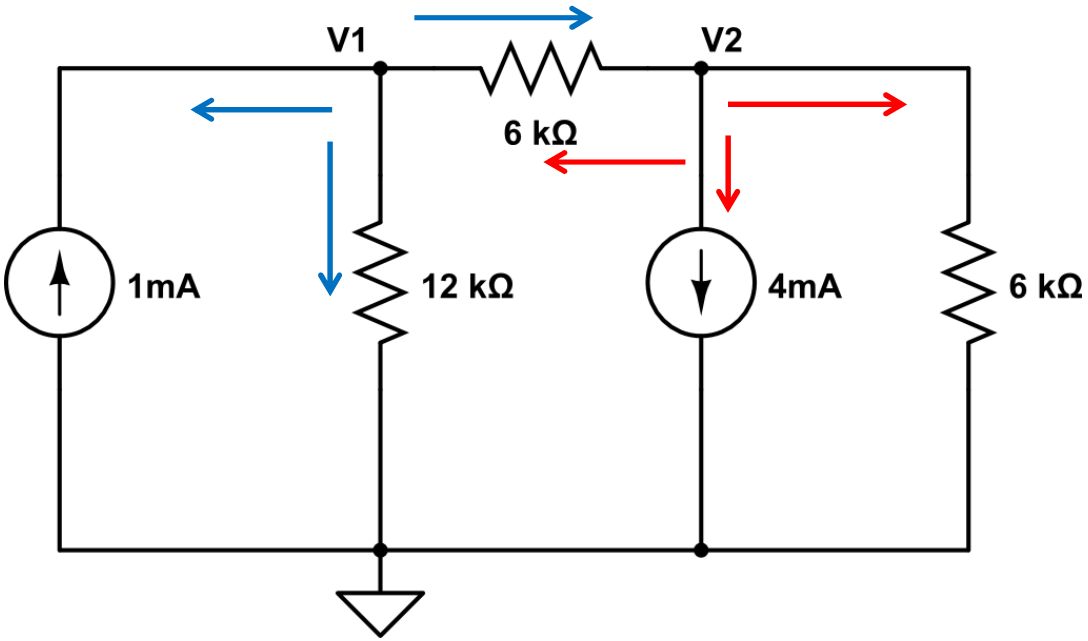
$$v_2 = 10.91V$$

Exercise

- Find the two node voltage equations that would allow us to find v_1 and v_2.



Solution



Node with v1:

$$-1mA+\frac{v_1}{12k}+\frac{v_1-v_2}{6k}=0$$

Node with v2:

$$\frac{v_2-v_1}{6k}+4mA+\frac{v_2}{6k}+=0$$

$$v_1\left(\frac{1}{12k}+\frac{1}{6k}\right)-v_2\left(\frac{1}{6k}\right)=1mA$$

$$v_1\left(-\frac{1}{6k}\right)+v_2\left(\frac{1}{6k}+\frac{1}{6k}\right)=-4mA$$

$$3v_1-2v_2=12V$$

$$-v_1+2v_2=-24V$$

$$v_1=2v_2+24$$

$$3(2v_2+24)-2v_2=12V$$

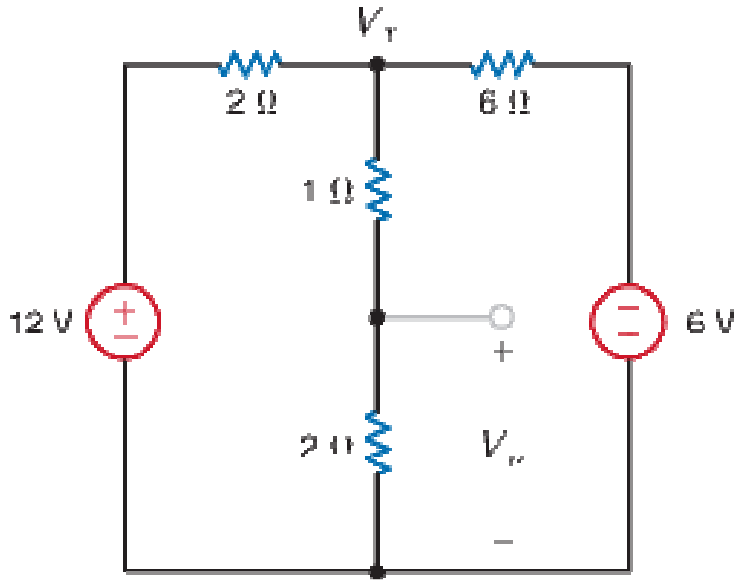
$$4v_2=-60V$$

$$v_2=-15V$$

$$3v_1+30=12V$$

$$v_1=-6V$$

Example FE Exam Problem



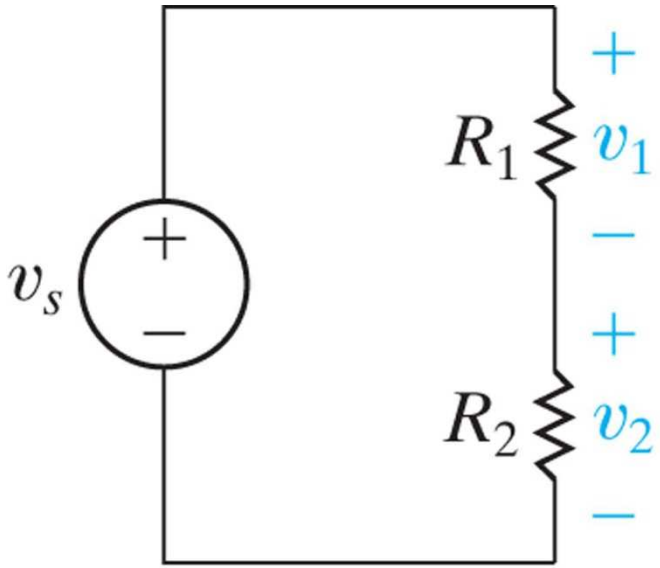
Find Vo in the circuit.

a) 3.33 V
b) 8.25 V
c) 9.33 V
d) 2.25 V

Additional Techniques

Voltage Divider

- The voltage divider is a simple way to find the voltage across two resistors in series.

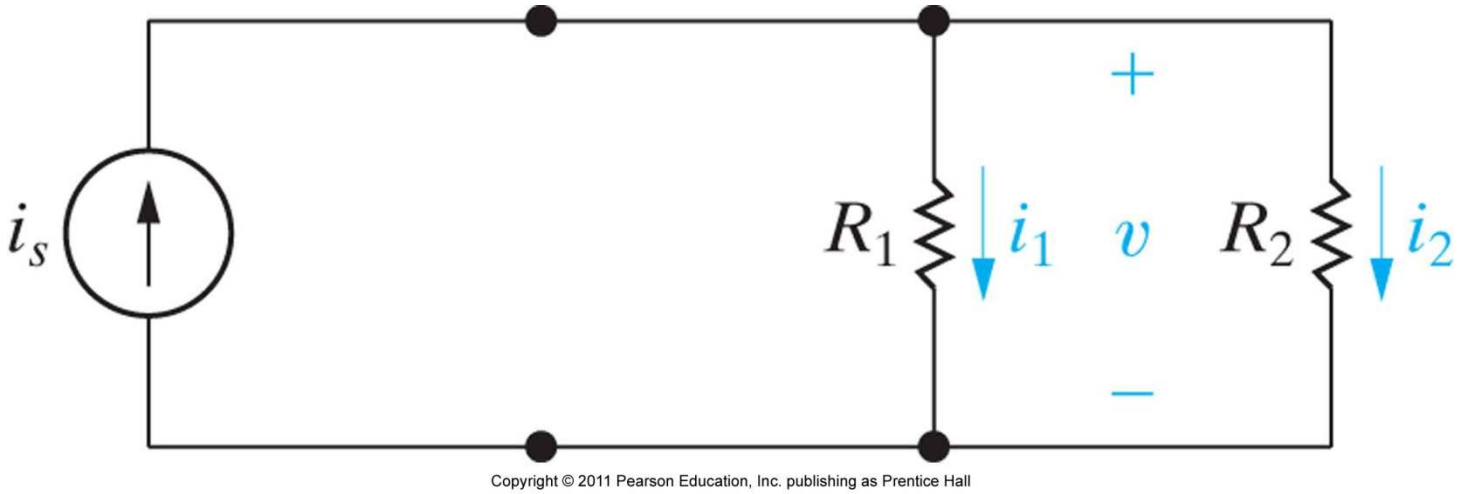


$$v_1 = v_s\left(\frac{R_1}{R_1 + R_2}\right)$$

$$v_2 = v_s\left(\frac{R_2}{R_1 + R_2}\right)$$

Current Divider

- The current divider allow you to find the current flowing through one of two parallel resistors



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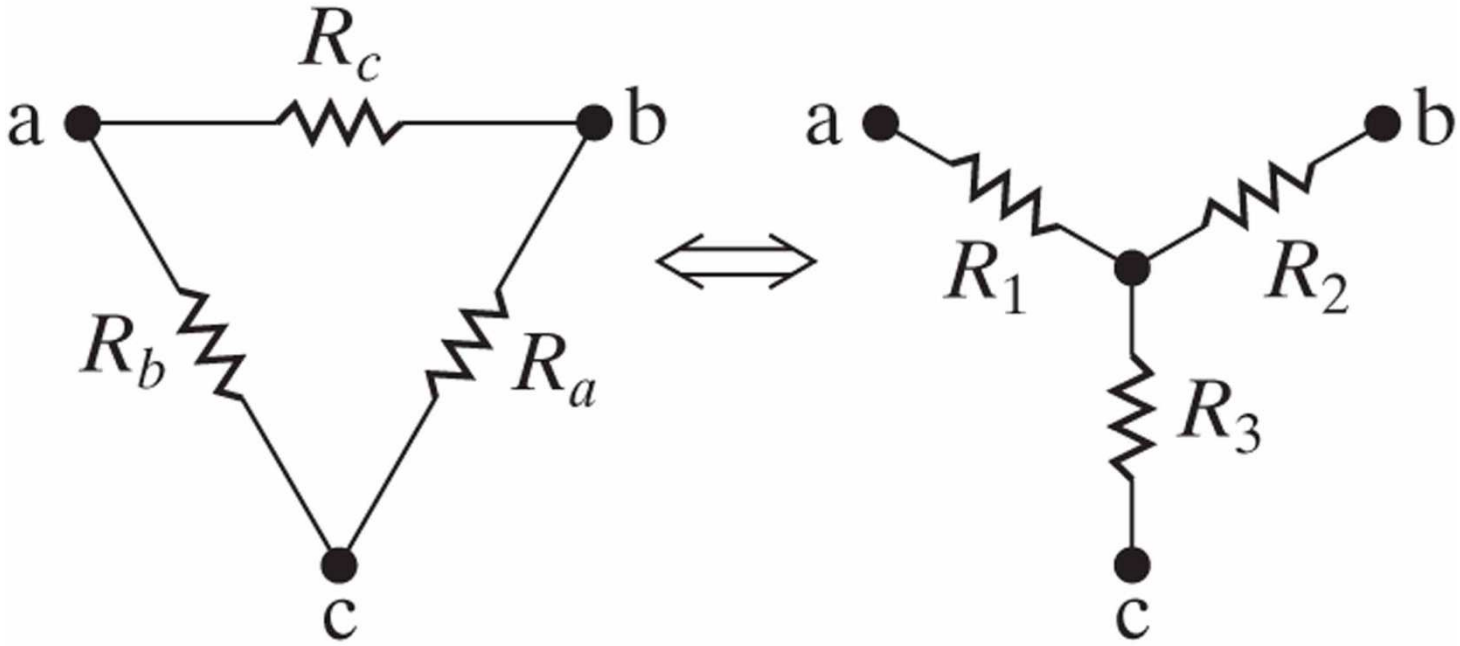
$$i_1 = i_s\left(\frac{R_2}{R_1 + R_2}\right) \qquad i_2 = i_s\left(\frac{R_1}{R_1 + R_2}\right)$$

Delta-Wye Transforms

- There is another simplification that can be made to make solving resistive circuits easier
- This is the Delta-Wye Transform



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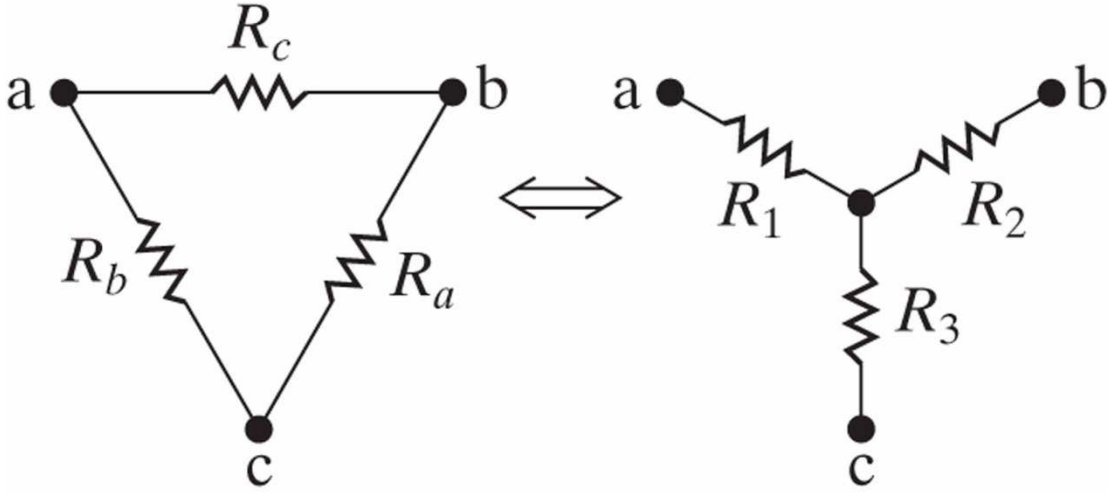
Transform Equations



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Delta to Y

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

Y to Delta

$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

$$R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

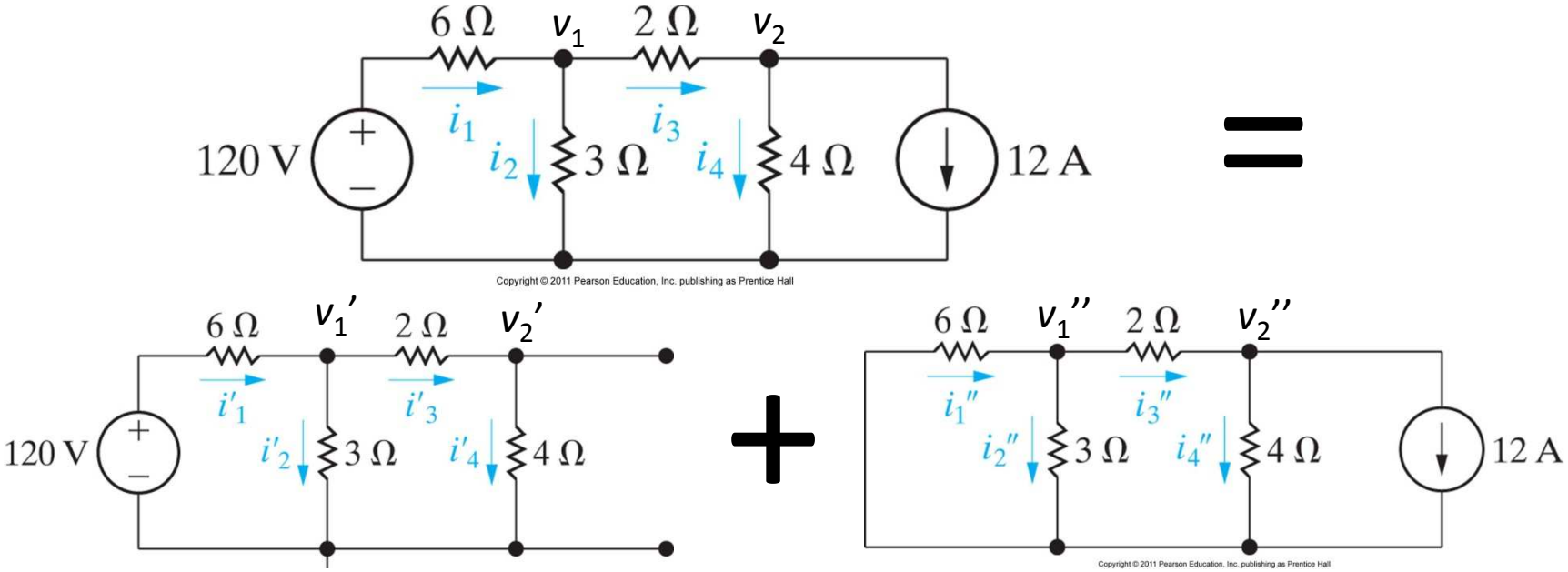
$$R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Superposition

- All linear systems obey the principle of superposition.
- This means that whenever a linear system is driven by multiple sources, the total response of the system is the sum of the responses to each individual source
- This principle is used in all areas of engineering and physics and is of course useful in Circuit Theory

What does this mean for solving circuits?

- The value of voltage or current relating to any element in a circuit can be thought of as the sum of the voltages and currents resulting on that element from each of the independent sources in the circuit individually.



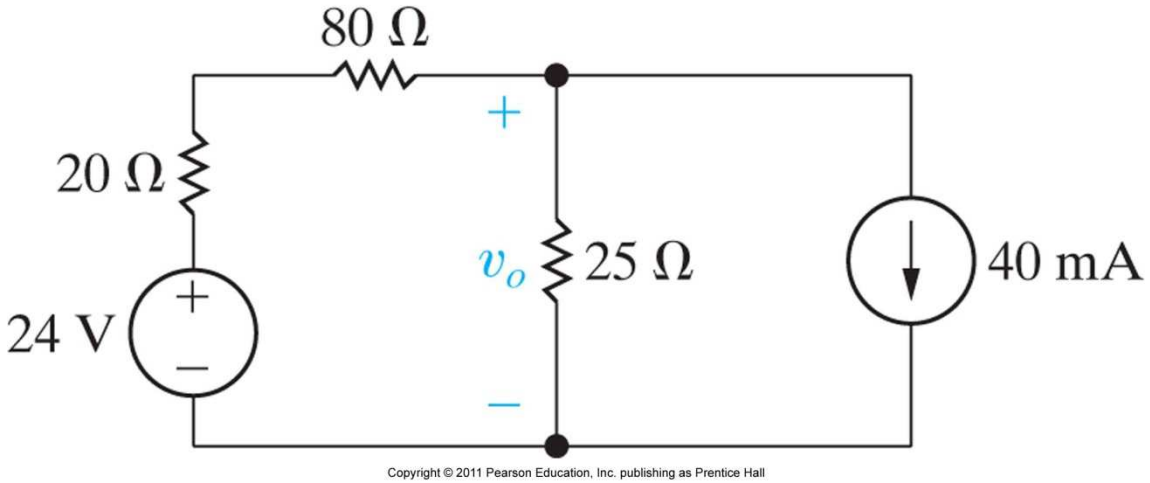
$$i_1 = i_1' + i_1'' \qquad i_3 = i_3' + i_3'' \qquad v_1 = v_1' + v_1''$$

$$i_2 = i_2' + i_2'' \qquad i_4 = i_4' + i_4'' \qquad v_2 = v_2' + v_2''$$

Uses for superposition

- Superposition is highly useful if you have a circuit that you have already solved that is altered by adding a new source of some type.
- All you need to do is deactivate all the original sources and find the values resulting from your new source
- The true altered values in your circuit are just the original values plus what results from the new source
- Also very useful when analyzing multiple signals in a system. You find found the output for each signal individually and then sum the results to find that full solution.

Example



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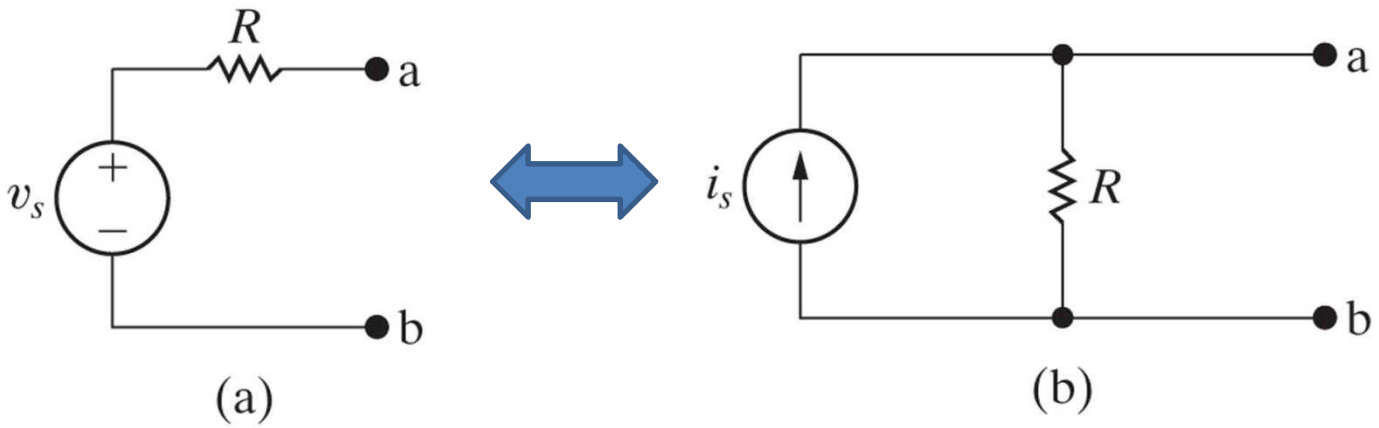
With only 24V source: $$v_o' = 24V\left(\frac{25}{20+80+25}\right) = 24V\left(\frac{25}{125}\right) = 4.8V$$

With only 40mA source: $$v_o'' = -40mA(25\,//\,100) = -40mA\left(\frac{25(100)}{25+100}\right) = -0.8V$$

$$v_o = v_o' + v_o'' = 4.8 - 0.8 = 4V$$

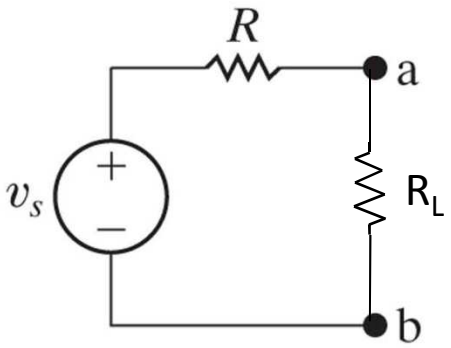
Source Transformations

- Another method like Delta-Wye and Parallel/Series combinations to simplify circuits
- Can switch between equivalent sources: a voltage source with series resistor or current source with parallel resistor



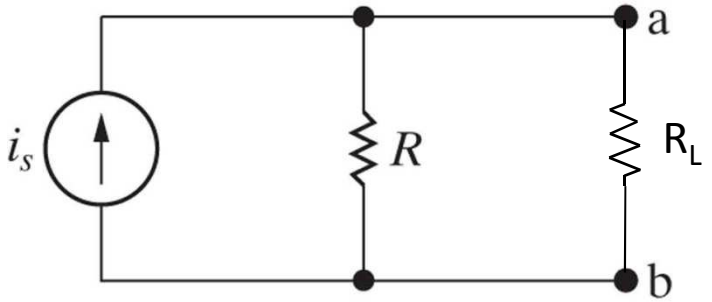
How to Transform

- Each of the two equivalent sources must produce the same current through any load connected across the source.



(a)

$$I_{load} = \frac{v_s}{R + R_L}$$



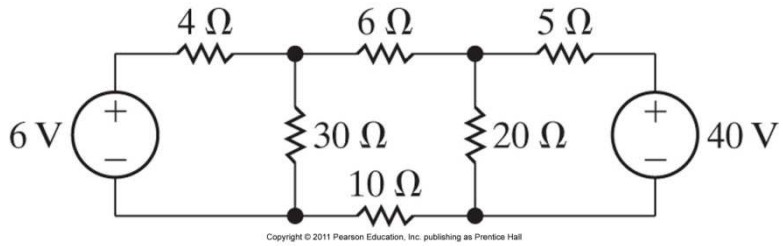
(b)

$$I_{load} = i_s \frac{R}{R + R_L}$$ (current divider)

$$\frac{v_s}{R + R_L} = i_s \frac{R}{R + R_L}$$ ⇒ $$v_s = i_s R \quad \text{OR} \quad i_s = \frac{v_s}{R}$$

Source Transform Example

What is Power of 6V source?



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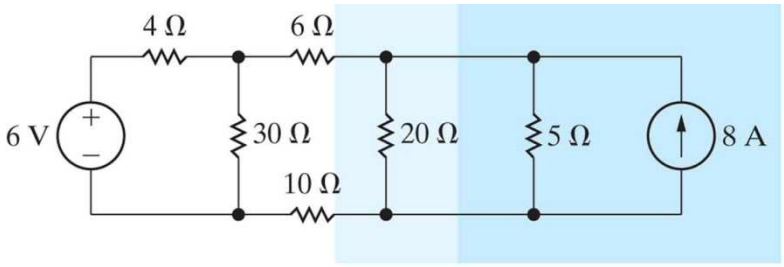
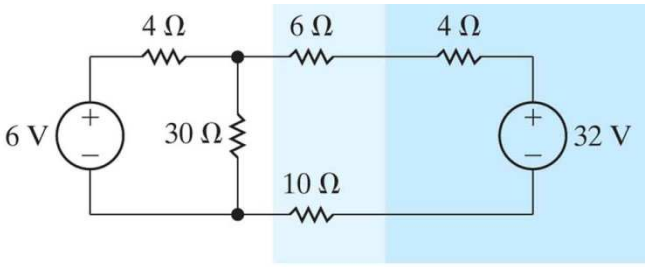
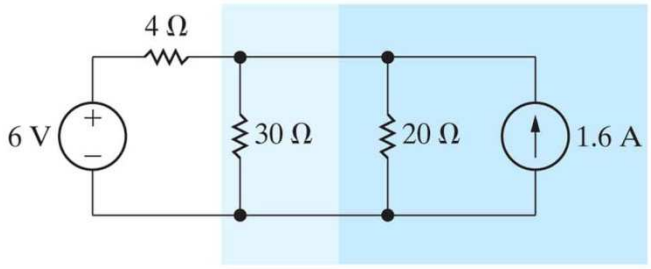


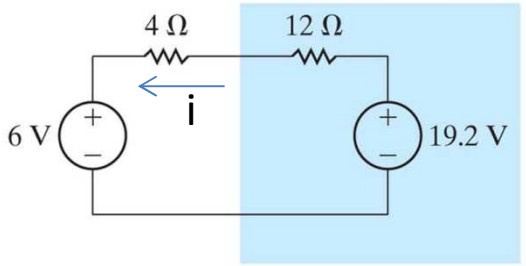
(a) First step



(b) Second step



(c) Third step



(d) Fourth step

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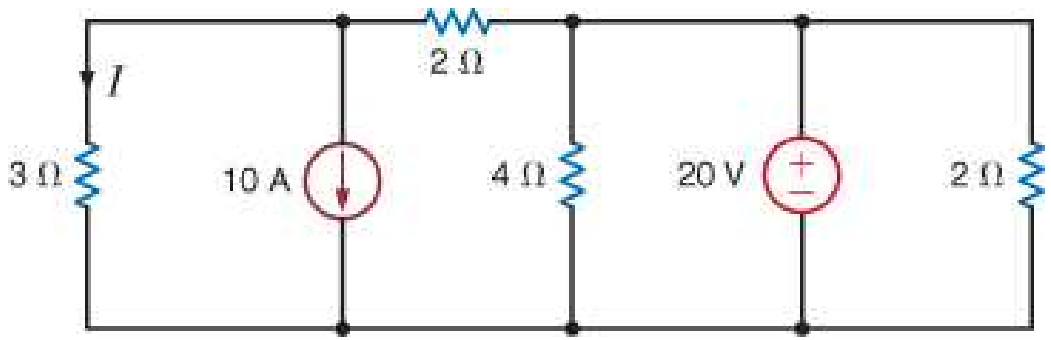
i = (19.2V-6V)/16 = 0.825A

P = i*v = 0.825A*6V = 4.95W

Example FE Exam Problem



Find *I* in the circuit.

a) 8 A
b) -4 A
c) 0 A
d) 4 A

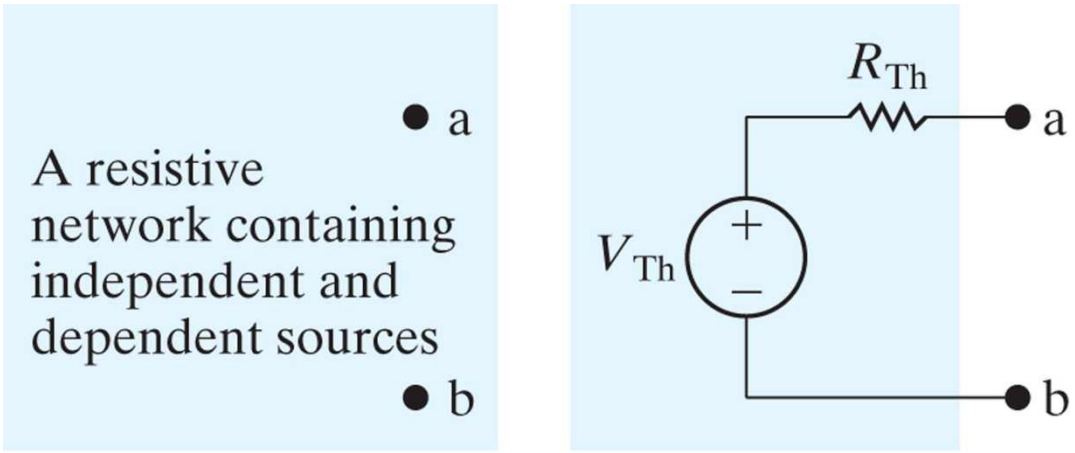
Thevenin Equivalent Circuits

Terminal Behavior

- We are often interested in looking at the behavior of an electrical circuit at it's output terminals.
- If a load is hooked to a circuit, how does the circuit behave relative to the load?
- If the circuit is made of linear elements, the entire circuit can be simplified into a Thevenin (or Norton) Equivalent circuit.

Justification

- We can consider the entire circuit a black box with two output terminals. The behavior at these terminals can be modeled by a voltage source and a series resistor.



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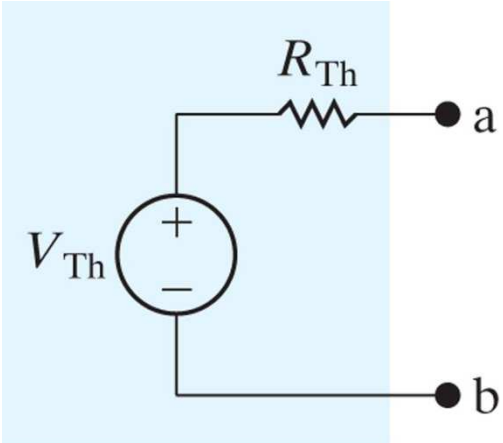
Finding the Thevenin Voltage

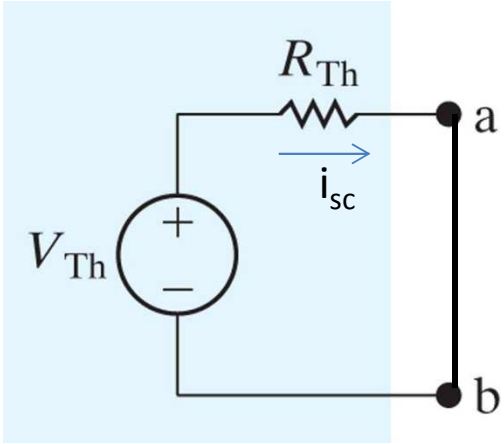
- If we look at the terminal with no load attached (open circuit output), the voltage between the output terminals is V_{Th}.



With an open circuit, no current flows
This means no voltage drop across R_{Th}
Hence $V_{ab} = V_{Th}$

Finding Thevinen Resistance

- Now lets look at what happens when the output is short circuited



$$i_{sc} = \frac{V_{Th}}{R_{Th}}$$

$$R_{Th} = \frac{V_{Th}}{i_{sc}}$$

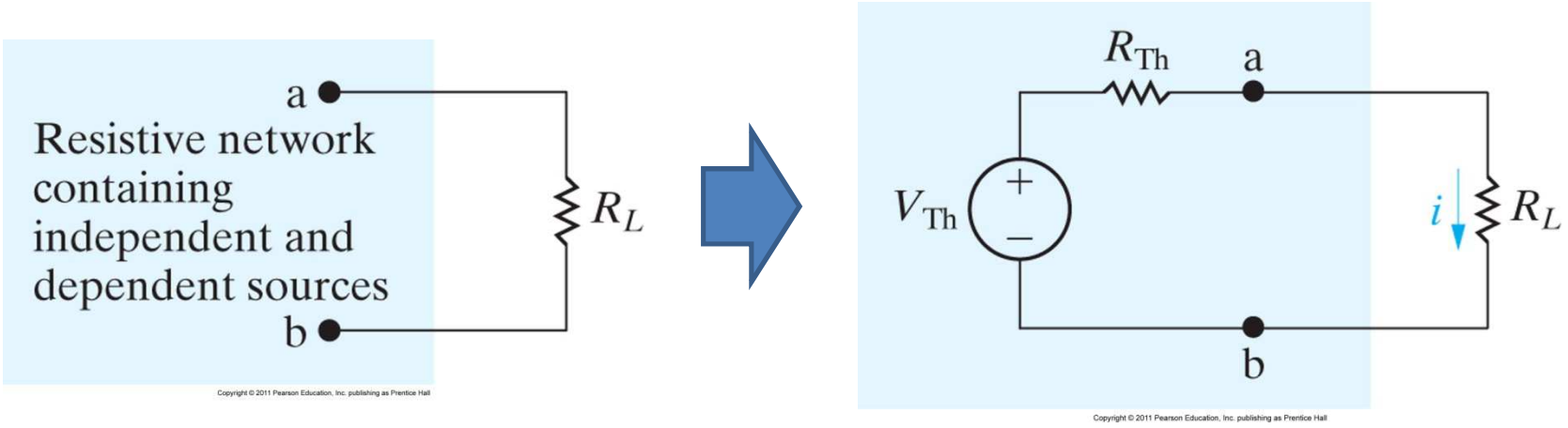
- If we can find the short circuit current and have already found V_{Th}, then we can easily calculate R_{Th}

Power Transfer

- We are often concerned with maximizing the amount of power we transfer from one circuit to another
- In Power systems the primary concern is efficiently transferring power to the load. Losses must be kept to a minimum.
- In communication systems we often want to transfer data from a transmitter to a detector. Optimizing the power that is transferred to a receiver or other load circuit is highly important

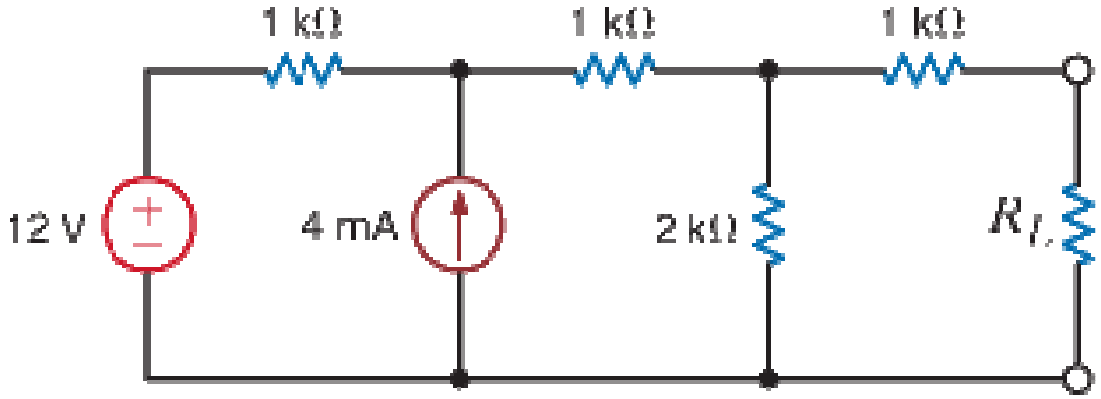
Maximum Power Transfer

- We have some network of sources and linear elements and will connect this network to a load R_L. We want to maximize power in the load.



- **The power transfer is maximized when R_L is equal to R_{TH}**

Example FE Problem



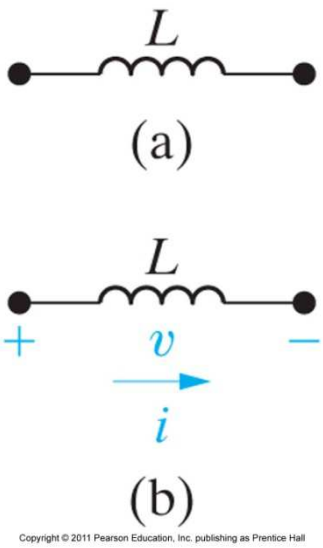
Determine the maximum power that can be delivered to the load resistor R_L in the circuit shown.

a) 2 mW
b) 10 mW
c) 4 mW
d) 8 mW

RL and RC Circuits

Inductors

- Inductors are magnetic coils the produce an induced voltage proportional to the rate of change of current in the coil



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Current and Voltage Relations

$$v(t) = L\frac{di}{dt}$$

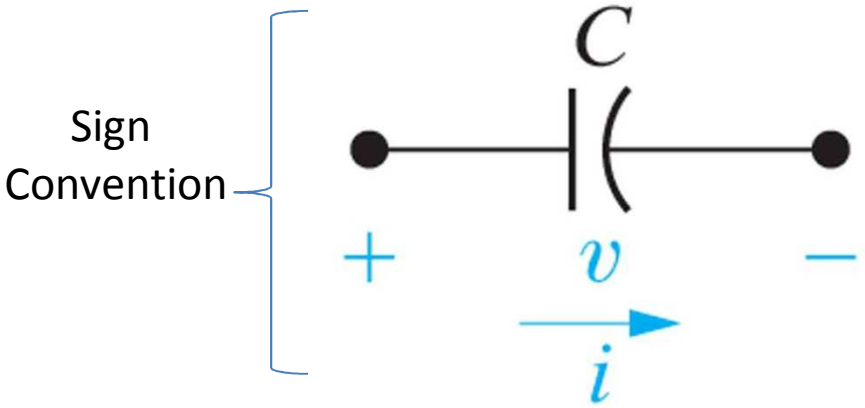
$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

Power as a function of time $P(t) = Li\frac{di}{dt}$

Energy Stored in the inductor $w = \frac{1}{2}Li^2$

The Capacitor

- A capacitor is a circuit component represented by the letter C
- Capacitance is measured in Farads
  - Typical values are in the pF to μF range
  - A 1F capacitor is extremely large
- The symbol for a capacitor is shown below:



Capacitor Current and Voltage Relations

- The current and voltage relations for a capacitor are opposite to those of the inductor

Inductor:

$$v(t) = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$$

Capacitor:

$$i(t) = C\frac{dv}{dt}$$

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$$

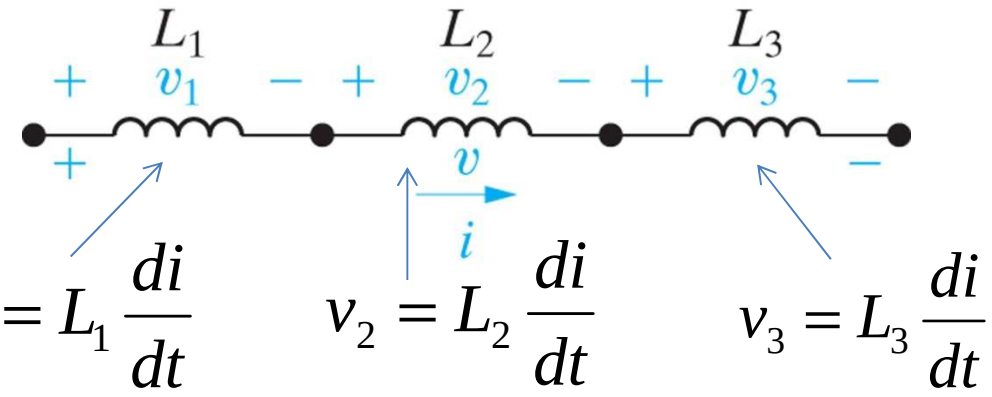
Power and Energy Relations

- Similarly, the power and energy storage relationships for a capacitor are reversed in terms of voltage and current from those of the inductor:

	Inductor	**Capacitor**
Power as a function of time	$P(t) = Li\frac{di}{dt}$	$P(t) = Cv\frac{dv}{dt}$
Stored Energy	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}Cv^2$

Series and Parallel Combinations of Inductors and Capacitors

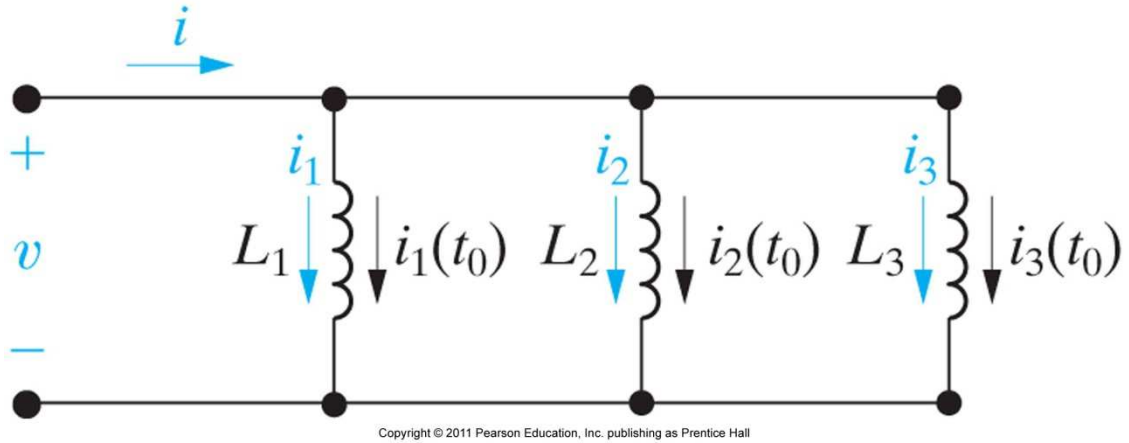
- **Inductors add in series much like resistors:**



$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3 + L_4 + \ldots + L_n$$

Parallel Inductors



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$$i = i_1 + i_2 + i_3$$

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t} v\,d\tau + i_1(t_0) \qquad i_2 = \frac{1}{L_2}\int_{t_0}^{t} v\,d\tau + i_2(t_0) \qquad i_3 = \frac{1}{L_3}\int_{t_0}^{t} v\,d\tau + i_3(t_0)$$

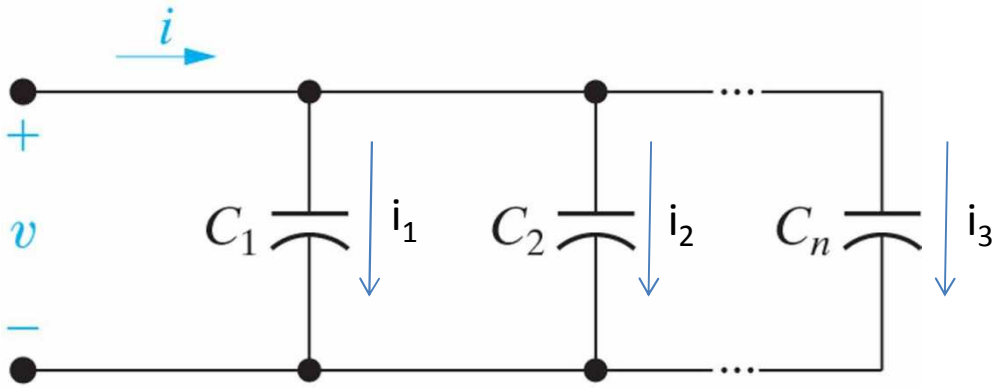
$$i = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v\,d\tau + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4} + \ldots + \frac{1}{L_n}\right)^{-1}$$

$$i(t_0) = i_1(t_0) + i_2(t_0) + \ldots + i_n(t_0)$$

Combinations of Capacitors

- **Capacitors add when in parallel:**



$$i = i_1 + i_2 + i_3$$

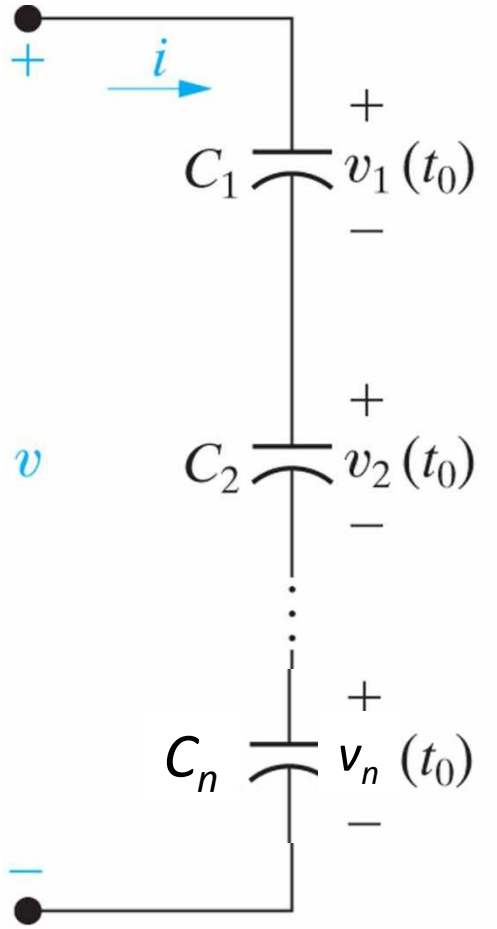
$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt} \qquad i_3 = C_3 \frac{dv}{dt}$$

$$i = i_1 + i_2 + i_3 = (C_1 + C_2 + C_3)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + \ldots + C_n$$

Series Combinations of Capacitors

Inductors add in the inverse when in parallel, just like resistors



$$v_1 = \frac{1}{C_1}\int_{t_0}^{t} i\,d\tau + v_1(t_0)$$

$$v_2 = \frac{1}{C_2}\int_{t_0}^{t} i\,d\tau + v_2(t_0)$$

$$v_n = \frac{1}{C_n}\int_{t_0}^{t} i\,d\tau + v_n(t_0)$$

$$v = v_1 + v_2 + \ldots + v_n$$

$$v = \left(\frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}\right)\int_{t_0}^{t} i\,d\tau + v_1(t_0) + v_2(t_0) + \ldots + v_n(t_0)$$

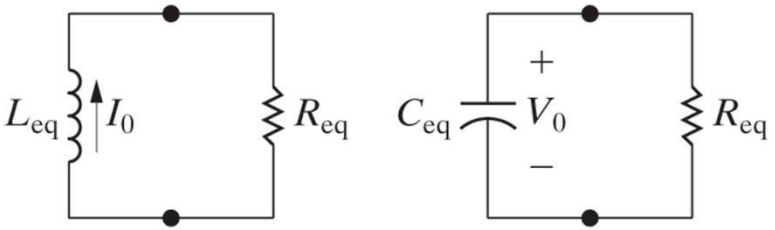
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n}$$

$$v(t_0) = v_1(t_0) + v_2(t_0) + \ldots + v_n(t_0)$$

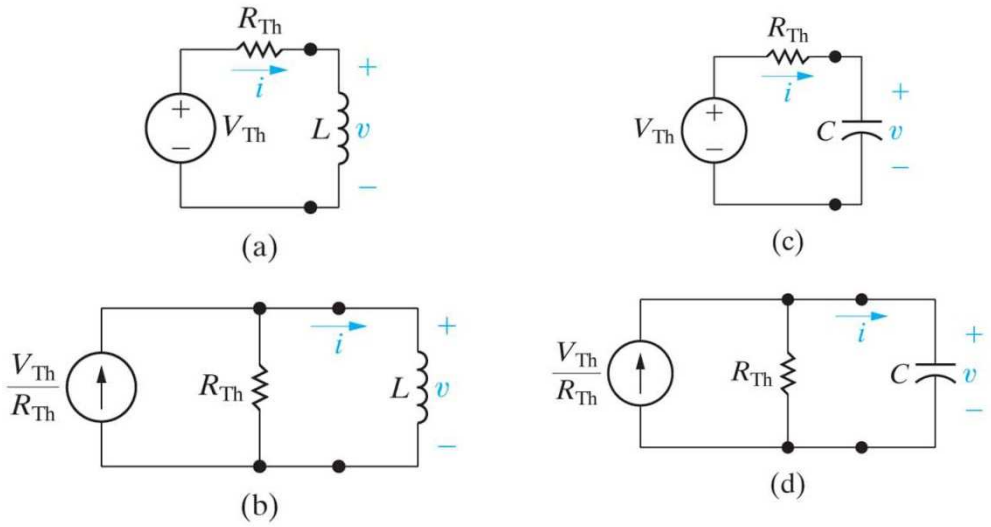
RC and RL Circuit Transients

- Inductors and capacitors can be charged or discharged through a resistive element

Discharge



Charge



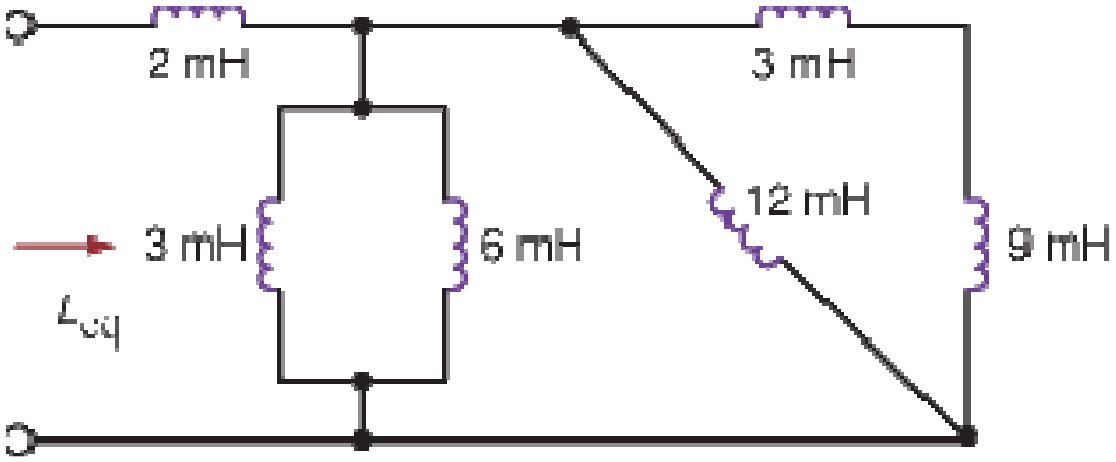
General Form of RC and RL Response

- All off these simple RC and RL circuits have the same form of response:

$$X(t) = X_{final} + \left(X_{initial} - X_{final}\right)e^{\frac{-(t-t_0)}{\tau}}$$

- X can be current or voltage in either the inductor or capacitor
- Tau is the time constant:
  - τ=RC for RC circuit
  - τ=L/R for RL circuit

Example FE problem

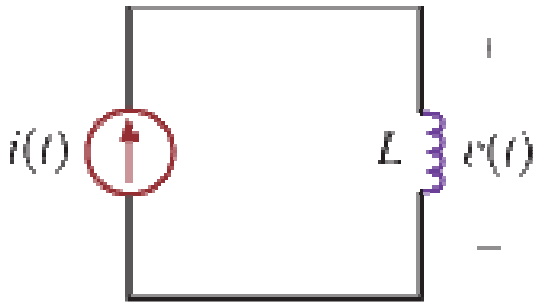


What is the equivalent inductance of the circuit?

a) 9.5 mH
b) 2.5 mH
c) 6.5 mH
d) 3.5 mH

Example Problem #2

What is the voltage across the 10 mH inductor expressed as a function of time?



$$i(t) = \begin{cases} 0\,A, & t < 0 \\ 20\,te^{-2t}, & A > 0 \end{cases}$$

a) $$v(t) = \begin{cases} 0\,V, & t < 0 \\ -2te^{-2t}V, & t > 0 \end{cases}$$

b) $$v(t) = \begin{cases} 0\,V, & t < 0 \\ 2e^{-2t} + 4te^{-2\,t}V, & t > 0 \end{cases}$$

c) $$v(t) = \begin{cases} 0\,V, & t < 0 \\ -0.2te^{-2t} + 0.4te^{-2\,t}V, & t > 0 \end{cases}$$

d) $$v(t) = \begin{cases} 0\,V, & t < 0 \\ 0.2e^{-2t} - 4te^{-2\,t}V, & t > 0 \end{cases}$$

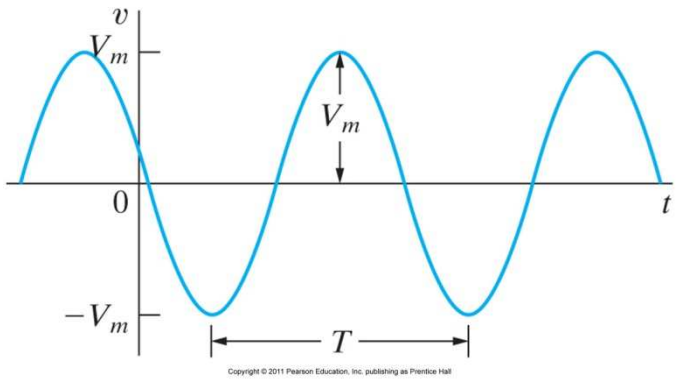
AC Circuits and Analysis

AC Waveforms

- AC waveforms are sinusoidal and can be described by either a sine or cosine

$$v(t) = V_m \cos(\omega t + \phi)$$

V_m = Amplitude
ω = angular frequency (equal to 2πf or 2π/T)
ϕ = phase constant



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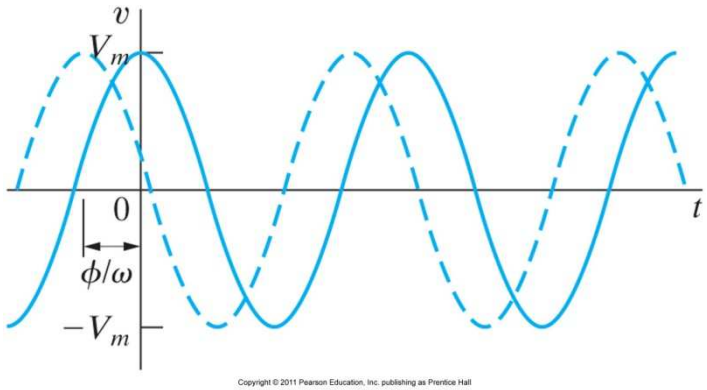

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Sine-Cosine Relations

- The following are useful relationships for AC circuits:

$$\cos(\omega t) = \sin(\omega t + \frac{\pi}{2}) = -\sin(\omega t - \frac{\pi}{2})$$

$$\sin(\omega t) = \cos(\omega t - \frac{\pi}{2}) = -\cos(\omega t + \frac{\pi}{2})$$

RMS Values

- AC sinusoids have an average of zero, so we use RMS (root of the mean squared function) to get a measure for effective average value.

$$V_{rms} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} \left(V_m \cos\left(\omega t + \phi\right)\right)^2 dt}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Phasors and Phasor Transform

- We use Euler's Identity to express sinusoidal functions in the complex –number domain.

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$ → Euler's Identity

$$v = V_m \cos(\omega t + \phi) = V_m \Re\left\{e^{j(\omega t+\phi)}\right\} = V_m \Re\left\{e^{j\omega t} e^{j\phi}\right\}$$

The Phasor Representation or Phasor Transform of a Sinusoidal waveform:

$$\mathbf{V} = V_m e^{j\phi} = P\left\{V_m \cos(\omega t + \phi)\right\}$$

Impedance

- Impedance (Z) is the complex analog to simple resistance.

$$\mathbf{V} = Z\mathbf{I}$$

Ohm's Law for AC Circuits

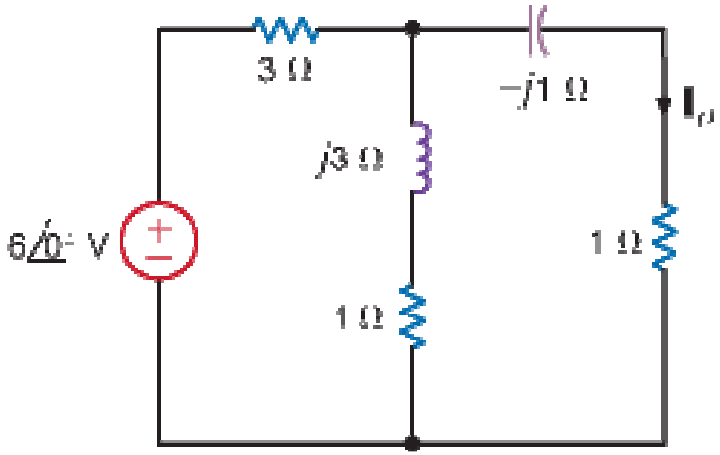
Circuit Element	Impedance	Reactance
Resistor	R	-
Inductor	jωL	ωL
Capacitor	-j/ωC	-1/ωC

Reactance is the imaginary part of the Impedance

Kirchhoff's Laws with AC circuits

- All of Kirchhoff's Laws still hold with the phasor transform
  - Just sum complex currents and voltages in either the polar or rectangular form
- Impedances simply sum when in series and sum in the inverse when in parallel (just like resistors)

FE exam problem



What is the current Io in the circuit?

a) $6.32 \angle 30.31^\circ$ A

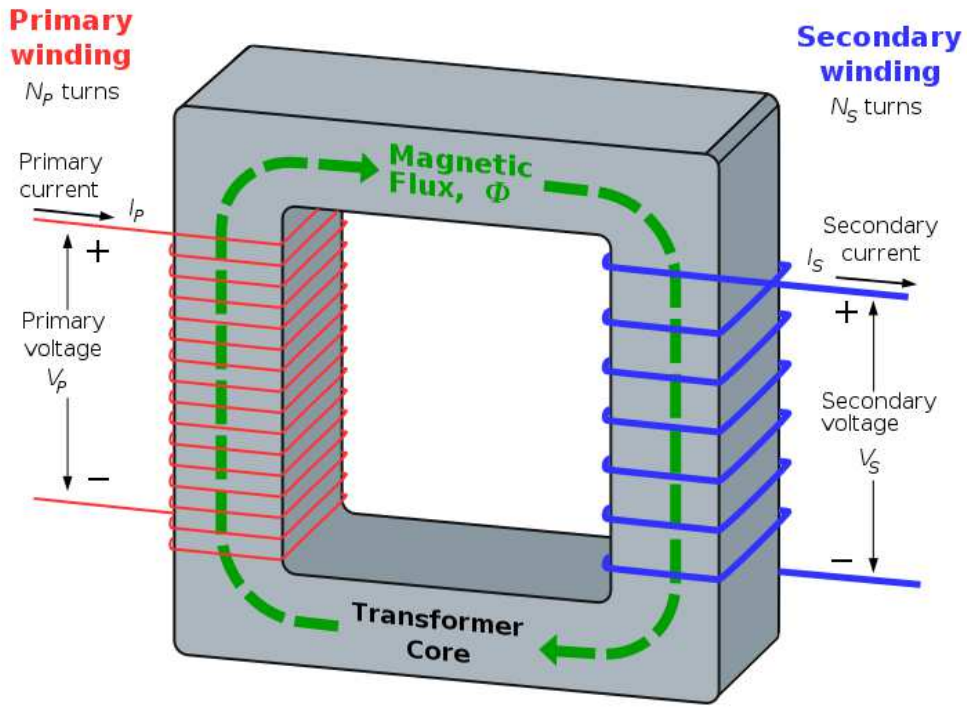
b) $2.75 \angle 21.43^\circ$ A

c) $1.48 \angle 32.92^\circ$ A

d) $5.23 \angle 40.15^\circ$ A

Transformers

- Two coils are wound on a single core to ensure magnetic coupling



Ideal Transformer

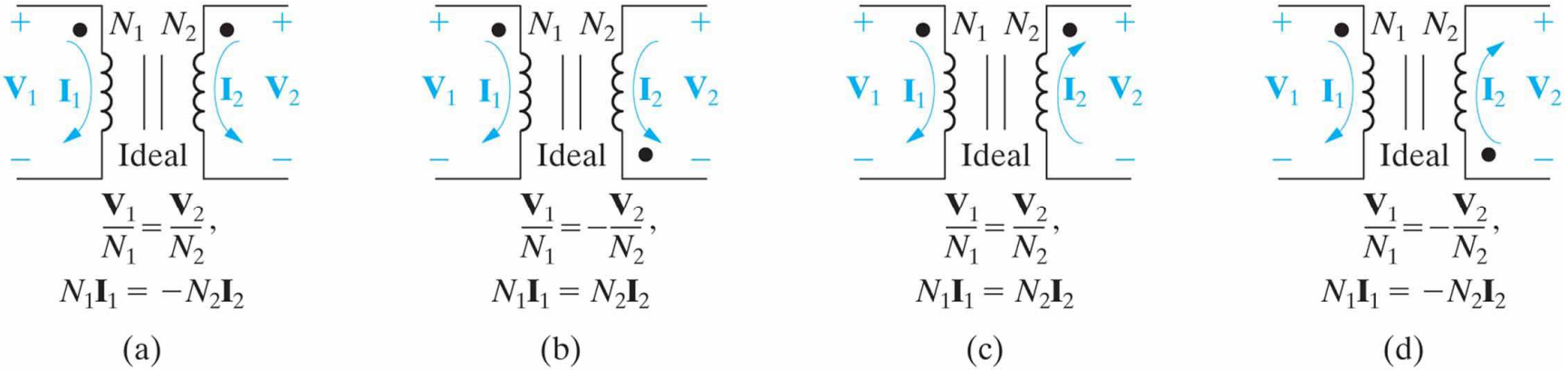
- For an ideal transformer the voltages and currents through each winding are related as shown below:

$$\frac{\mathbf{V}_1}{N_1} = \frac{\mathbf{V}_2}{N_2} \qquad \mathbf{I}_1 N_1 = \mathbf{I}_2 N_2$$

  - N_1=number of winding in coil 1
  - N_2=number of winding in coil 2
    - V_1= voltage across coil 1
    - V_2= voltage across coil 2
    - I_1 = Current through coil 1
    - I_2 = Current through coil 2

Voltage Polarity in an Ideal Transformer

The four circuits below show the dot convention and the relative voltage polarity and direction of current



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Voltage that is positive at a dotted terminal results in a voltage that is positive at the other dotted terminal

Current flowing into a dotted terminal should result in current flowing out of the other dotted terminal

AC POWER

- Instantaneous Power = v(t)*i(t)
- =P+Pcos(2ωt)+Qsin(2ωt)
- P=Average Power= $\frac{V_m I_m}{2}\cos(\phi_v - \phi_i)$
- Q=Reactive Power= $\frac{V_m I_m}{2}\sin(\phi_v - \phi_i)$

Complex Power

- We have discussed Real Power (P) and Reactive Power (Q)
- These can be thought of as the real and imaginary components of complex power (S)

TABLE 10.2 Three Power Quantities and Their Units

Quantity	Units
Complex power	volt-amps
Average power	watts
Reactive power	var

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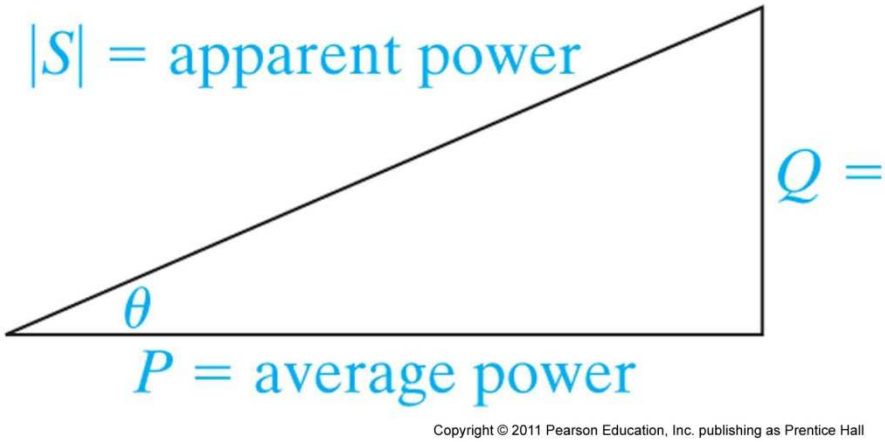
$$S = P + jQ$$

Apparent Power

- When plotted in the complex plane, Complex Power can be though of as the result of the addition of perpendicular P and Q.



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$$\theta = (\phi_v - \phi_i)$$

- The magnitude of the complex power |S| is known as Apparent Power

Meaning of Apparent Power

- Apparent Power is the amount of Power that must be generated in order to supply a given amount of average power
- If the circuit has reactive elements (which any power network will, such as power lines, generator windings, etc.) There will be a reactive element to the total power.
- If $|Q|>0$, then the amount of power that must be supplied to drive a circuit that is dissipating a certain amount of real power (P) is greater than P.
- Q can be balanced by compensating inductance with capacitance, making the power factor $[\cos(\phi_v-\phi_i)]=1$
- With proper balancing the amount of apparent power needed to provide a given average power to a load can be minimized

Expressions for Power

- Real Power: $P = I_{rms} V_{rms} \cos(\phi_v - \phi_i)$

$$P = \left|I_{rms}\right|^2 R = \frac{\left|V_{rms}\right|^2 R}{R^2 + X^2}$$

- Reactive Power: $Q = I_{rms} V_{rms} \sin(\phi_v - \phi_i)$

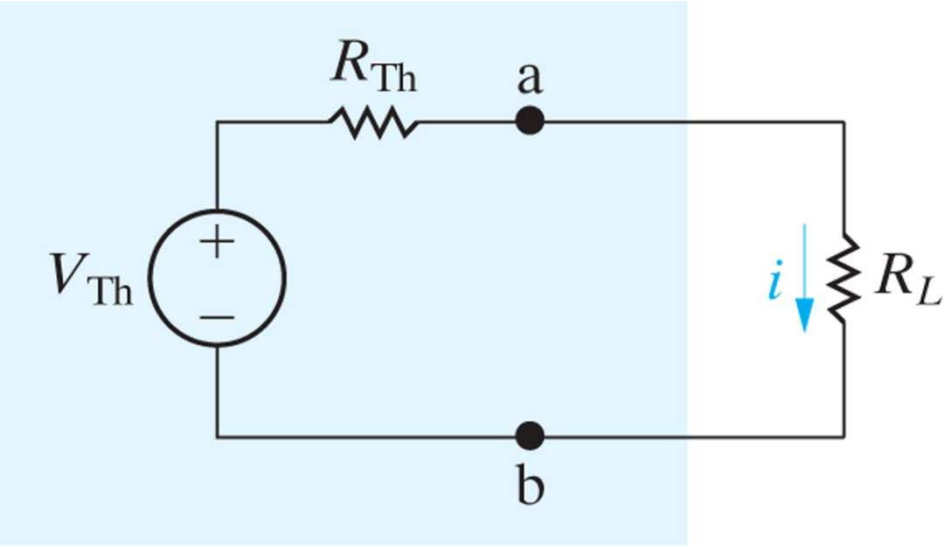
$$Q = \left|I_{rms}\right|^2 \mathrm{X} = \frac{\left|V_{rms}\right|^2 X}{R^2 + \mathrm{X}^2}$$

- Complex Power: $S = I_{rms}^{*} V_{rms}$

$$S = \left|I_{rms}\right|^2 Z = \frac{\left|V_{rms}\right|^2}{Z^*}$$

Maximizing Power Delivered to a Load

- Remember from resistive circuits that the maximum power delivered from a circuit to a load resistor occurred when the load resistance was equal to the Thevenin Resistance of the driving circuit.



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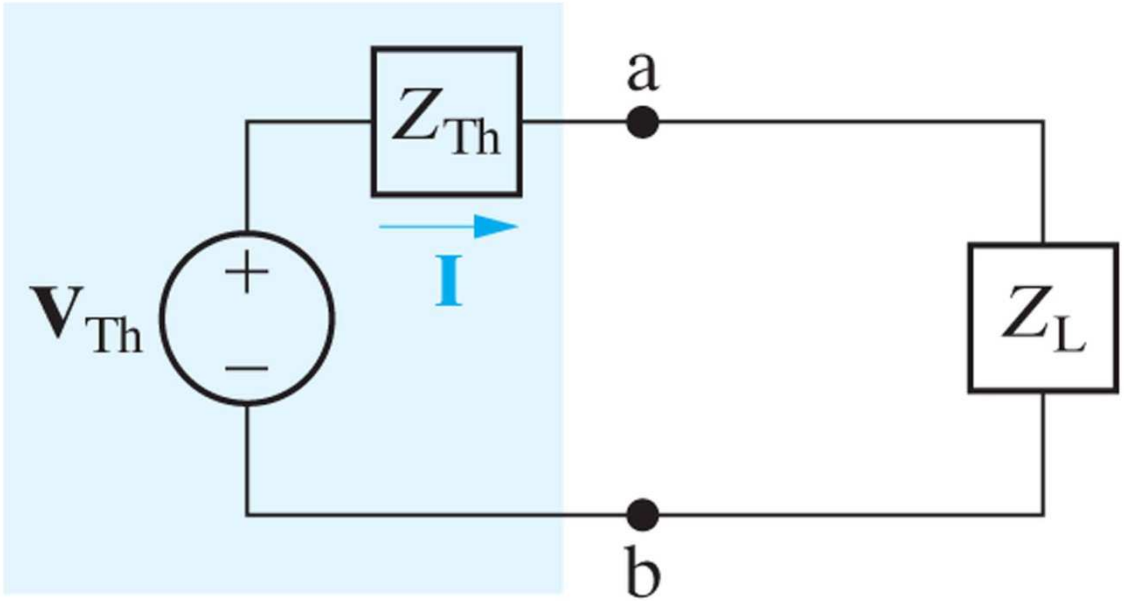
Maximizing Power Transfer with complex circuits



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With circuits containing reactive elements, maximum power is transferred when the load impedance is equal to the complex conjugate of the Thevenin Equivalent Impedance

$$Z_L = Z_{Th}^*$$

$$P_{\max} = \frac{1}{4}\frac{\left|V_{Th}\right|^2}{R_L}$$

Explanation

- Look at a complex impedance in series with it's complex conjugate:

$$Z_{Th} = A + jB$$

$$Z_{Th}^{*} = A - jB$$

$$Z_{Th} + Z_{Th}^{*} = A + jB + A - jB = 2A$$

- The reactive elements will cancel, causing the total power factor to equal 1. Each element has the same resistance resulting in maximum power transfer into the resistive part of the load (max real power)