LATIN HYPERCUBE SAMPLING: APPLICATION TO PIT LAKE HYDROLOGIC MODELING STUDY

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ABSTRACT

On behalf of a mining company, Shepherd Miller, Inc. (SMI) evaluated the hydrology of a lake that would ultimately form in an abandoned open mine pit. Using analytical modeling and best-estimate input parameters, SMI's initial evaluation indicated that the pit lake would be a terminal sink. Under these conditions, groundwater would flow towards the pit lake in all directions and water in the lake would not recharge the aquifer system. Thus, pit lake water would have no chemical impact on the regional groundwater system. Regulatory agencies were concerned that uncertainties associated with input parameters might precluded definitive conclusions regarding the groundwater impact evaluation.

Through sensitivity analysis, SMI identified input parameters that most influenced the modeling results. SMI then quantified the uncertainties associated with the parameters and incorporated these into the modeling effort using the Latin Hypercube (LHS) sampling technique. LHS is a method for performing uncertainty analysis similar to the Monte Carlo method. Based on the uncertainty analysis, SMI concluded that there was an 80% probability that the future pit lake would operate as a terminal sink to groundwater. The LHS approach addressed the concerns raised by the regulatory agencies without the need for additional site investigations or numerous modeling runs.

Key words: *pit lake, terminal sink, LHS sampling technique*

INTRODUCTION

As part of a proposed expansion, a mining company planned to develop a new open pit mine. The plan was to excavate an ore zone below an existing mountain, creating an open pit that would extend approximately 700 feet below original ground surface and nearly two hundred feet below the existing water table. Previous hydrogeologic baseline characterizations provided information on the local ground and surface water flow regimes. Using analytical models and best-estimate hydrologic input parameters, SMI predicted that a lake would form in the open pit after cessation of mining. The pit lake was predicted to be a terminal sink (that is, no water would flow from the lake into the groundwater system), have a depth of approximately 116 feet, and take about 40 years to reach 95% of its stable elevation (See Figure 1). To address regulatory agency concerns, SMI proceeded with an effort to quantify the uncertainties associated with these predictions

OBJECTIVES

The main objectives of the analysis were to quantify the uncertainties associated with the pit lake model predictions and to provide the regulatory agencies with estimates of the probability that the lake would operate as a terminal sink. A terminal pit lake poses potential water quality impacts to the lake water itself, but not to regional groundwater. A flow-through lake (that is, where groundwater flows into and out of the lake) may cause a potential chemical impact to both the pit lake water and down-gradient groundwater. Thus, a focus of the analysis was to quantify the uncertainty associated with the prediction that the lake would be terminal. Uncertainties associated with pit lake depth and evaporation losses were also evaluated.

APPROACH

In order to describe the uncertainty analysis, a general explanation of the pit lake hydrologic model is necessary. Figure 2 shows the conceptual water balance developed for the future pit lake. Yearly average values for each of the main inflow and outflow components were quantified from regional meteorological information, site characterization data, and/or professional judgment. A synopsis of the development of these components is described in the following paragraphs.

The evaporation and precipitation components of the pit lake water balance were estimated from yearly average values multiplied by the lake surface area. Statistical evaluation of annual data indicated relatively small variances from the average values. Thus, precipitation and evaporation were each assigned constant (average) values that were carried through the subsequent uncertainty analyses.

Precipitation falling on the pit walls may either evaporate from the benches or reach the pit lake through surface water runoff or shallow bedrock recharge. There is some uncertainty associated with the fraction of pit wall precipitation that actually enters the pit lake. Based on experience at similar mine sites, this fraction ("excess pit wall precipitation") was assigned an average value equal to 15% of the mean annual precipitation.

During site characterization, short-term aquifer tests and borehole pumping tests were conducted to measure the *in situ* hydraulic conductivity of geologic materials near the proposed pit. The geometric mean (best-estimate value) of the measured hydraulic conductivities was 4.53×10^{-6} cm/s.

Groundwater inflow to the mine pit was determined to be a major component of the pit lake water balance. Figure 3 shows the steady state analytical flow model used to predict pit inflows. A detailed description of the analytical model is provided in Marinelli and Niccoli (1998). Flow conditions within Zone 1 are described by the following steady state equation:

$$h_o = \sqrt{h_p^2 + \frac{W}{K_1}\left[r_o^2 \ln\left(\frac{r_o}{r_p}\right) - \frac{r_o^2 - r_p^2}{2}\right]}$$

where W is the distributed recharge flux; K_1 is the hydraulic conductivity of materials within Zone 1; r_p is the effective pit radius; h_p is the saturated thickness above the base of Zone 1 at r_p; r_o is the radius of influence (maximum extent of the cone of depression); and h_o is the initial (pre-mining) saturated thickness above the base of Zone 1. Given the values of W, K_1, r_p, h_p, and , the radius of influence (r_o) is determined from the above equation by iteration. Once r_o is determined, the pit inflow rate through the pit walls (Zone 1) is computed by:

$$Q_1 = W\pi\left(r_o^2 - r_p^2\right)$$

Groundwater flowing through the pit bottom (Zone 2) is estimated from the following steady state equation:

$$Q_2 = 4\ K_2\ r_p\left(h_o - d\right)$$

where Q_2 is the inflow rate; K_2 is the hydraulic conductivity of materials within Zone 2; and d is the depth of the pit lake. In all calculations, it was assumed that Zone 1 and Zone 2 were characterized by the same hydraulic conductivity (K) value.

A water balance model for the future pit lake was developed based on the expected pit geometry, the groundwater inflow equations, and assumed hydrologic input parameters. The model assumed that at any point in time, total inflows minus outflows were equal to the rate change in the pit lake water volume. The rate of rise in the pit lake water level was then equal to the rate change in water volume divided by the pit lake surface area. The pit lake depth over time was predicted by assuming a succession of steady state water balance conditions over short time increments. Sensitivity analyses, performed on the water balance, indicated that the model was most sensitive to values of hydraulic conductivity, effective pit radius, and the excess pit wall precipitation factor. These three parameters were evaluated in the uncertainty analysis described below.

UNCERTAINTY ANALYSIS

To address the uncertainty in selecting input parameter values, SMI chose the Latin Hypercube Sampling (LHS) technique described by McKay et al. (1979). LHS is similar in concept to the Monte Carlo approach for addressing uncertainty. The Monte Carlo technique is based on simple random sampling (SRS) of the input variables and generally requires a large number of realizations (simulations) to be statistically meaningful. In contrast, the LHS is based on a combination of SRS and stratified sampling techniques that leads to statistically significant results with substantially fewer realizations. Compared to the Monte Carlo approach, LHS produces an unbiased estimate of the mean and a probability distribution function of the model output, while creating a smaller variance. The variance reduction of LHS translates to fewer model simulations necessary to obtain the same degree of precision as SRS without data stratification. To generate probability distributions of pit lake depths, evaporative losses, and terminal or flow-through conditions, the LHS was performed using the following steps:

1. Assign an inverse cumulative distribution function (cdf) for each input variable. The inverse cdf is needed to back-calculate the parameter value for each randomly chosen cdf value.
2. Choose the number of simulations (N) to be performed.
3. Divide the cdf for each variable into N equi-probable intervals.

4. For each interval, choose a random sample, x, from the inverse cdf and develop a data set for each parameter. (This data set will reproduce the cdf of each parameter.)
5. Randomly select from each parameter input data set to create N model input sets.
6. Use an analytical or numerical model to determine a realization for each model input set. Based on the results of N model runs, develop a cumulative probability function for the model realizations.

These steps are illustrated by the hypothetical example shown on Figure 4.

RESULTS AND DISCUSSION

The inverse cdf based on the hydraulic conductivity data from testing is shown on Figure 5. Because effective pit radius and excess pit wall precipitation were not measured values, the respective cdfs were assumed to be linear over the range of expected values of these variables.

LHS was performed on each of the three cdfs; an input data set was generated for each parameter; and N model input sets (Table 1) were created by randomly selecting from each data set. The water balance model was used to determine a realization for each model input set presented in Table 1.

Based on only 20 realizations, the cumulative probability for pit lake depth is presented in Figure 6. As shown, there is essentially a 100% certainty that a pit lake will form (that is, nearly 100% probability that the pit lake will have a depth greater than 2.75 feet). Furthermore, there is a 70% probability that the pit lake will be over 100 feet deep.

In order to determine if the pit lake would be flow-through or terminal, SMI defined a term called "Terminality." This term is based on an analytical solution for flow to a circular lake situated in an aquifer that, prior to lake development, had uniform flow field. For this condition, a net groundwater extraction rate (Q^*) can be determined for the case where the lake just becomes a terminal sink. Terminality is defined as:

$$T = \frac{2 * \left(Q_{out} - Q_{in} - Q^{*}\right)}{\left(Q_{out} - Q_{in}\right) + Q^{*}}$$

where T is the terminality factor, and Q_{out} and Q_{in} are total outflows and inflows, respectively, from the pit lake water balance. Negative values of T indicate tendency toward a flow-through pit lake and positive values indicate tendency toward a terminal lake. Increasing positive values of T indicate a greater tendency for the pit lake to operate as a terminal sink. Figure 7 shows that there is a 20% probability that the pit lake will be flow-through, which corresponds to an 80% probability that the lake will be a terminal sink.

Figure 8 shows the uncertainty results for evaporative losses from the pit lake. The graph indicates that there is an 80% probability that the evaporative loss will be less than 160 gallons per minute (gpm).

CONCLUSIONS

Through the use of LHS, the uncertainty associated with the pit lake predictions was quantified. The most significant findings are that (1) a pit lake will certainly develop; (2) evaporative losses from the lake will likely be less than 160 gpm; and (3) there is an 80% probability that the lake will be a terminal sink. The LHS method allowed the above conclusions to be reached with a fraction of the computational effort that would be required to perform a standard Monte Carlo uncertainty analysis.

This uncertainty analysis can be used by the involved regulatory agencies to evaluate probable impacts to the hydrologic regime. In addition, it sets a foundation upon which future discussions concerning these predictions can be based.

REFERENCES

Marinelli, F., and W.L. Niccoli. 1998, pending. Simple Analytical Equations for Predicting Groundwater Inflows to an Open Mine Pit. Manuscript submitted to *Groundwater*.

McKay, M.D., R.J. Bechman, and W.J. Conover. 1979. A Comparison of Three Methods for Selecting Values of Input Variables in the Analysis of Output from a Computer Code. *Technometics*, vol. 21, no. 2, pp. 239-245.

Table 1. Model input sets for each realization.

Realization	Hydraulic Conductivity K (cm/sec)	Effective Pit Radius r_p (ft)	Excess Pit Wall Precipitation Factor
1	9.34E-07	542.1	0.023
2	5.11E-07	598.9	0.191
3	9.87E-06	127.0	0.121
4	4.71E-05	209.9	0.011
5	2.28E-05	799.0	0.142
6	1.23E-06	323.5	0.133
7	1.69E-05	349.5	0.156
8	2.85E-04	642.3	0.048
9	367E-05	273.4	0.039
10	4.11E-04	959.0	0.204
11	1.28E-04	248.2	0.051
12	2.25E-07	900.9	0.065
13	4.72E-06	310.9	0.164
14	2.01E-04	366.0	0.188
15	3.77E-07	422.5	0.080
16	2.86E-06	192.1	0.092
17	7.70E-05	430.9	0.070
18	3.69E-06	694.6	0.119
19	1.59E-06	497.7	0.107
20	7.17E-06	270.1	0.173

Figure 1. Pit lake water balance predicitons.

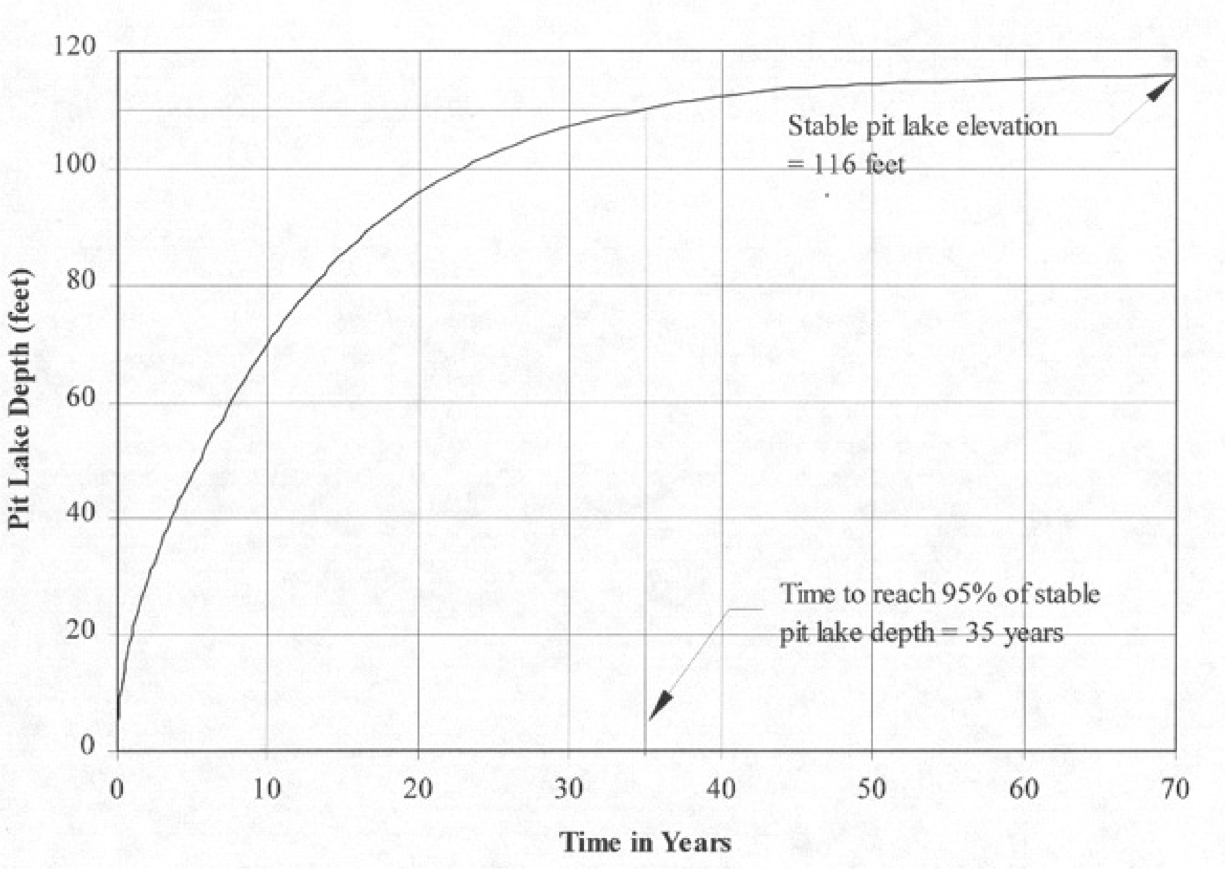

Figure 2. Conceptual water balance model of the pit.

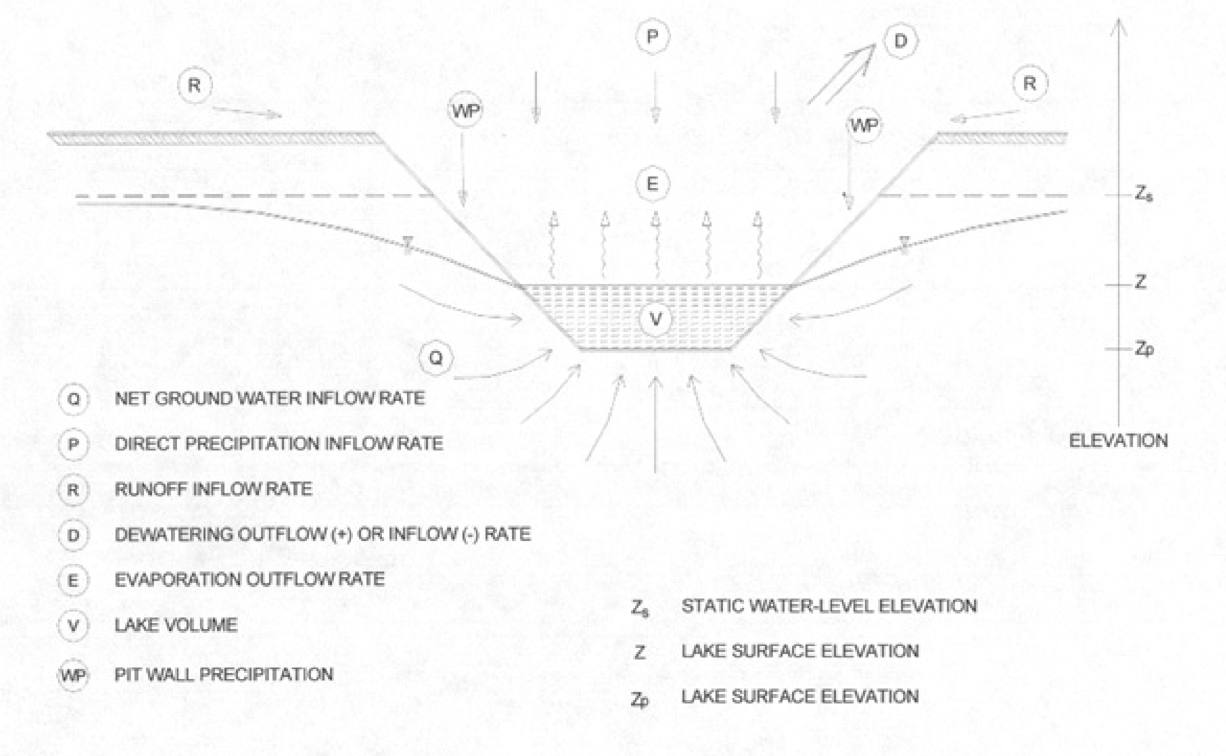

Figure 3. Analytical representation of groundwater flow to a pit lake.

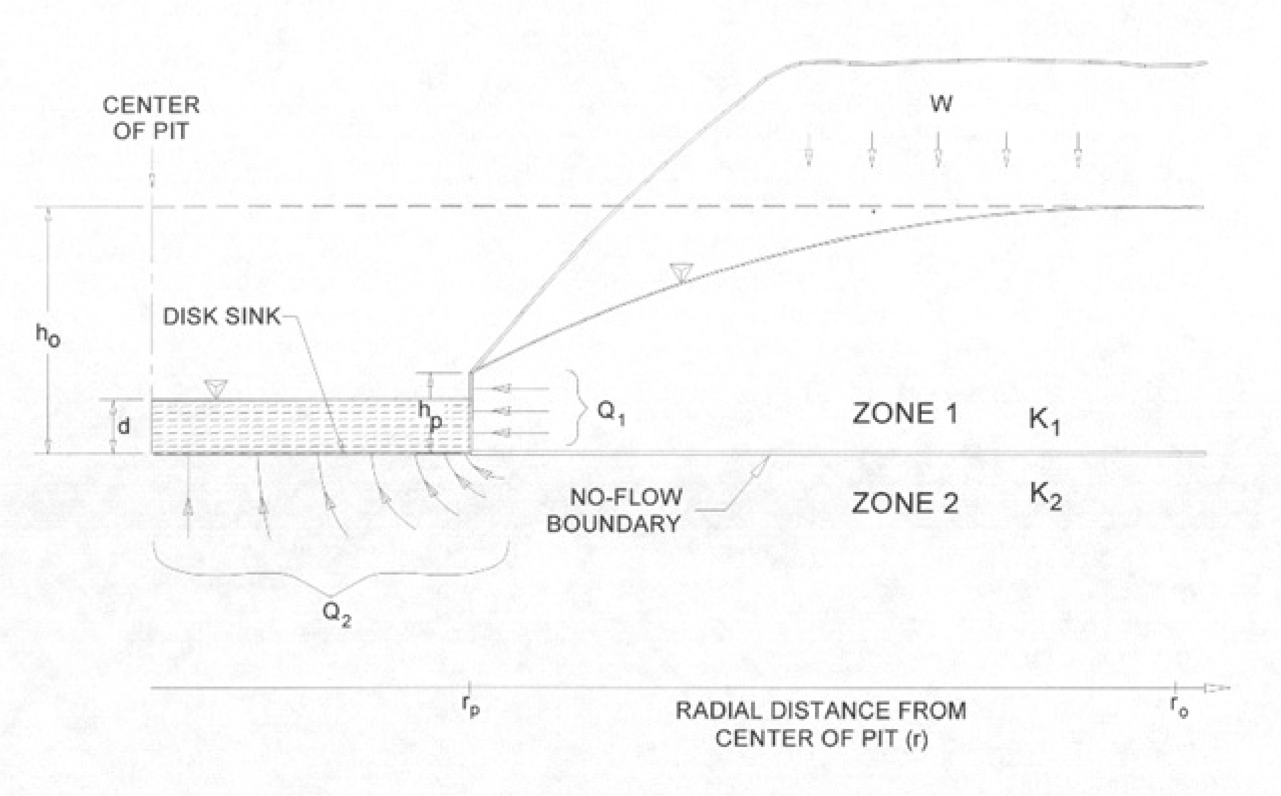

Figure 4. Example of the Latin Hypercube Sampling technique.

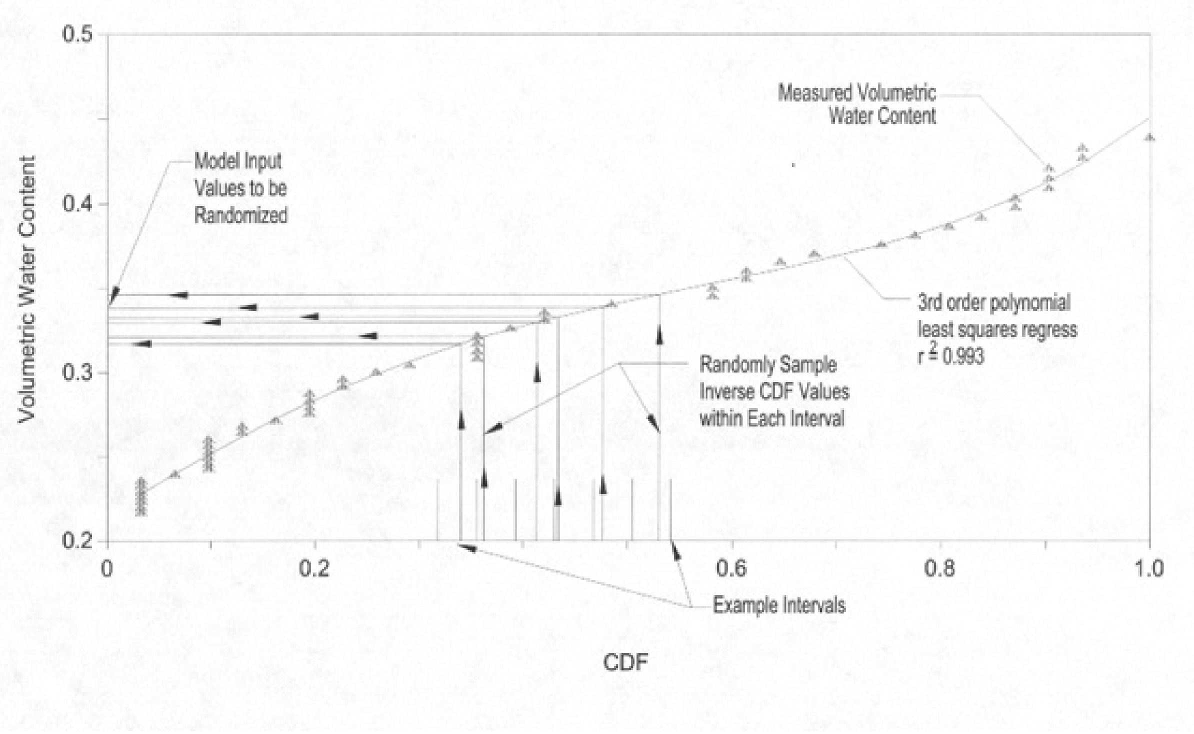

Figure 5. Inverse cumulative distribution function of hydraulic conductivity.

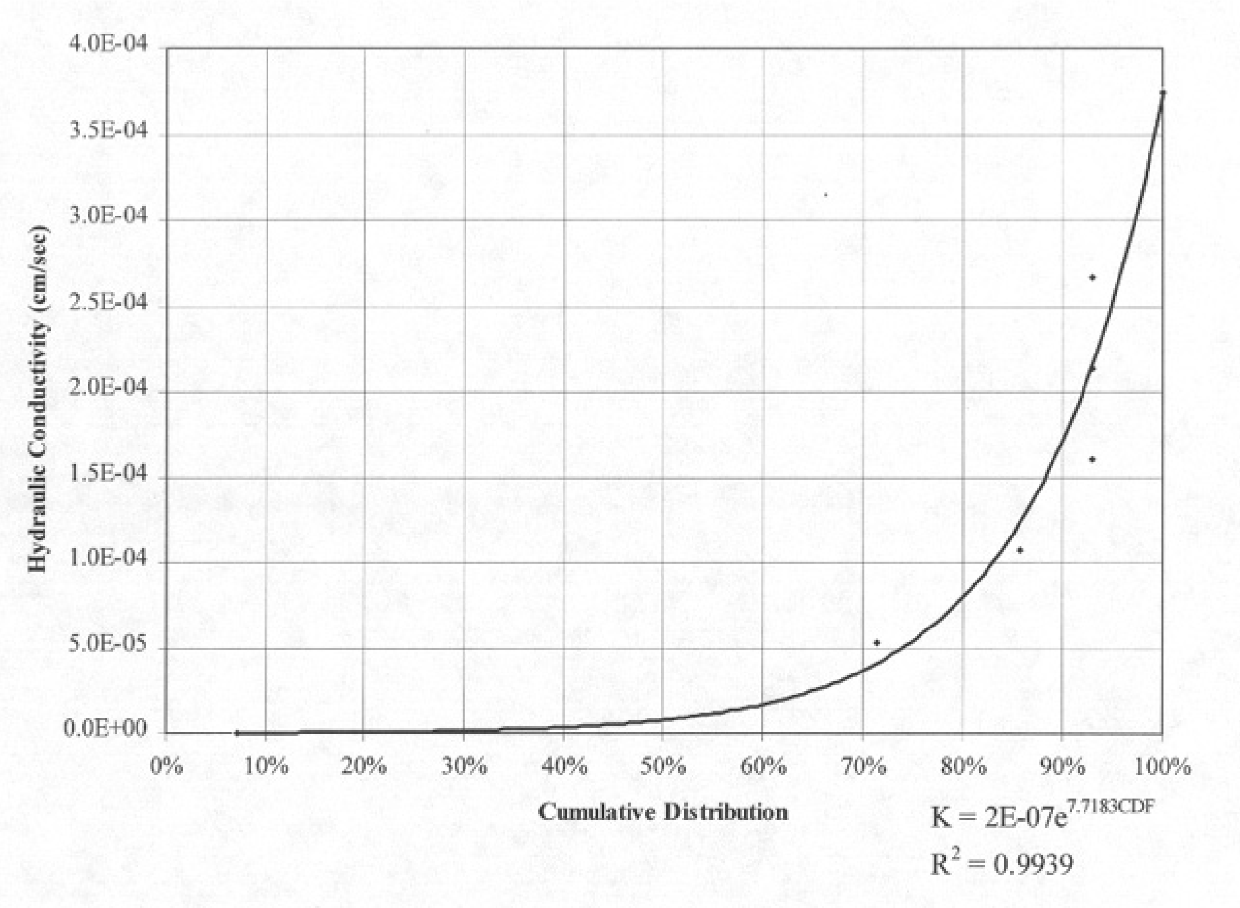

Figure 6. Uncertainty analysis results of pit lake depth predictions.

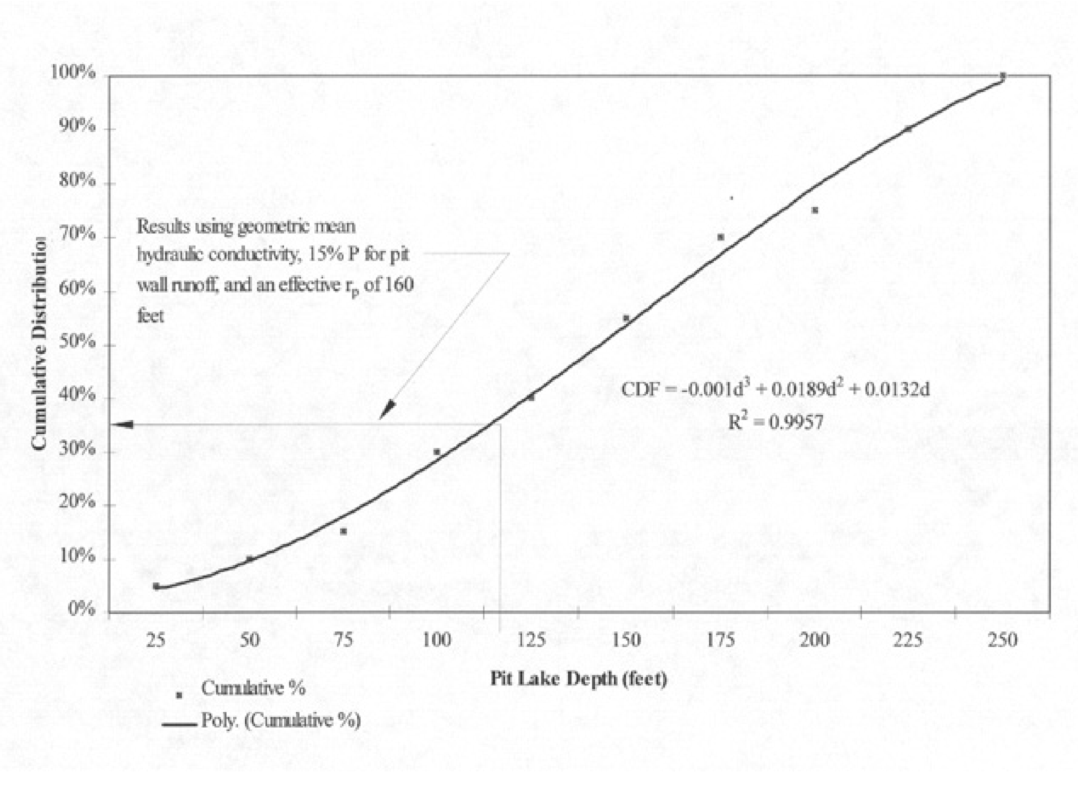

Figure 7. Uncertainty analysis results of terminal pit lake predictions.

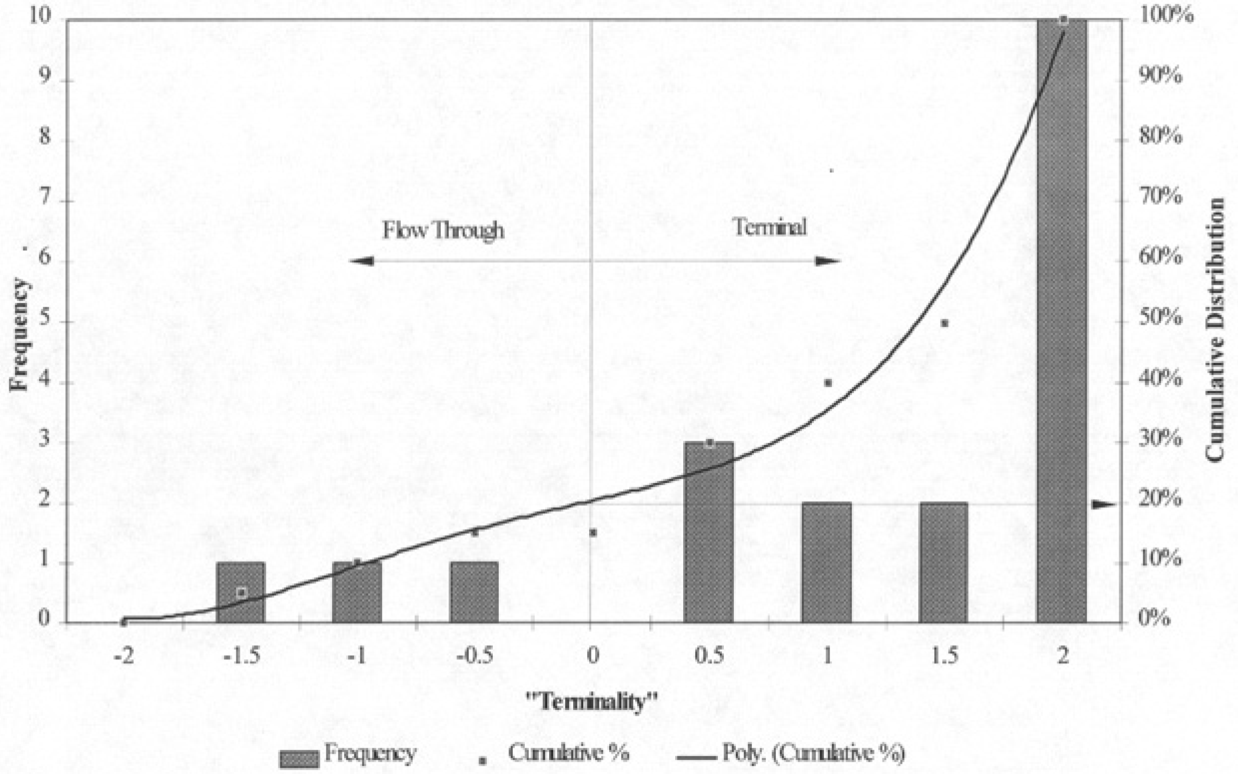

Figure 8. Uncertainty analysis results of evaporative losses from the pit lake.